USING OPEN ENDED TASKS WITH VISUAL REPRESENTATIONS IN CONNECTING LESSON STUDY WITH FORMATIVE ASSESSMENT

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1. INTRODUCTION

In some national curriculums stated that the mathematics communication is an important process that need to be developed beside the contents. The evaluation, exam also change to support the communication in the classroom: presentation, portfolio. The nature of classrooms requires communication. And communication is an important part of formative assessment. This is a professional skill of teachers of all subjects not only mathematics (speak, explain, prove...). But they do not use the term "communication" in the curriculum. Communication is considered as a professional skill of classroom teachers. Sometime students can solve the problems but feel difficult to explain, they use symbols, writing the solution.

To enhance the formative assessment effectively, the curriculum of some economies have innovations in assessments such as rubric assessment, clarification, representation, journal writing, visualization and model drawing. Some curriculums still emphasize in content knowledge but not pedagogy. The formative assessment needs to be improved.

In Vietnam, because the emphasis of old curriculum was on procedural knowledge and memorization of algorithms, students often worked independently completing exercises in the textbooks and workbooks. Students practiced mathematics independently to learn the mathematical concepts. When asking students questions, most teachers seek one “right” answer to the math problem and will explain why the answer is correct.

School reform mathematics education aims to help students achieve four following objectives: knowledge, skills, thinking and attitudes. Especially, with thinking objective, the mathematics curriculum provides opportunities for students to develop:

- The ability of observing, verifying, predicting, rational reasoning and logical reasoning;
- The ability of expressing precisely and clearly their own ideas and understanding the ideas of others;
- Spatial imagination;
- The characteristics of thinking, especially the flexible, independent and creative thinking.
- Thinking operations: comparison, analogy, generalization, and specialization.

The formative assessment of students' mathematical competencies by using open ended tasks with visual representations in lesson study provides evidence that they can express mathematical ideas by speaking, writing, demonstrating and depicting them visually.

2. USING OPEN-ENDED TASKS IN LESSON STUDY

In Vietnam, we are seeking for the innovation of teaching and learning mathematics. The teacher ought to think of teaching in terms of several principal hands-on activities with visual representations, problematic real life situations, and open-ended tasks. The innovation of teaching is to help students construct their own knowledge in an active way; enhance their thinking through solving non-routine problems while working cooperatively with classmates so that their talents and mathematical competencies are developed. There are several possibilities for innovation of mathematics education in an economy. Lesson study which originated from Japan is currently a central focus in US and other economies for the professional development of teachers and the improvement of students' mathematical competencies. In this session we will describe some open-ended tasks that were used in project on lesson study in Vietnam.

Example 1 (Grade 5): Find an appropriate decimal and fill in the blank such that: $0.1 < \ldots < 0.2$?

S1: $0.1 < 0.11 < 0.2$

T: Can you explain the reason why you choose 0.11?

S2: The appropriate decimals may be 0.11, 0.12, and 0.19.

T: Can you find another appropriate decimals?

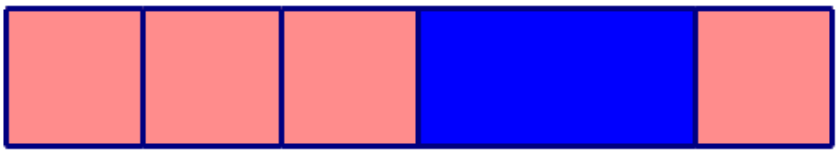
S3: 0.101, 0.102, …, 0.199.

T: How many appropriate decimals can you find between two decimal numbers?

S4: Many. For example, 0.1001, 0.1002, …

With this kind of teaching, teacher helps students dig deeply into a textbook problem and build up a habit of unsatisfying with achieved results, encourage students to be interested in seeking for another solutions, and creative in learning mathematics.

Example 2 (Grade 5)*:* Use 2 cm-cards and 4 cm-cards to make a toy train of 5 wagons?



This task is an open-ended task that requires pupils to make many trains as possible. Pupils can arrange the cards to make a train, use the strategy "*guess and check*" to get many answers. To solve this task mathematically teacher guides students to make a systematic list of all abilities.

N. of red wagons	0	1	2	3	4	5
N. of blue wagons	5	4	3	2	1	0
The length of the train in cm	20	18	16	14	12	10

From the above table, students recognize the relationship between the length and the numbers of red wagons, blue wagons. If the number of red wagons increases one, then the length of the train decreases 2 cm. In this task, students know that:

N. of red wagons + N. of blue wagons = 5

There are 6 options for this task. If the length of the train is given then we can find exactly the N. of red wagons and N. of blue wagons. The length of the train is understood as a restricted condition. Students will see that the train has the longest length 20 cm when all of the wagons are blue and shortest length 10 cm when all of the wagons are red.

The aim of this task is to help students recognize the restricted condition in finding two numbers that their sum is known.

Open-ended questions such as these are very useful for learning activities, and also as items in assessment instruments. There are multiple solutions and methods for solving them, so they are particularly useful in all classrooms as students have different levels of skill and understanding of concepts. They can be used to find out what students know about a concept when beginning a topic, investigating a concept, applying a skill, or ascertaining students' understanding or skills at the end of a teaching sequence. Since there are multiple solutions and methods for finding solutions to these kinds of questions, a student's solution and method of solution may be located on the continuum of standards used in state curriculum and assessment documents.

3. USING VISUAL REPRESENTATIONS IN LESSON STUDY

Representation involves decoding, encoding and interpreting familiar and less familiar representations of mathematical objects; choosing and switching between different forms of representation of mathematical objects and situations, and translating and distinguishing between different forms of representation. It further involves the creative combination of representations and the invention of non-standard ones.

This session emphasizes some of the positive effects of visualizing in mathematical concept formation and to show how visual representations can be used to achieve more than just a basic, procedural and mechanical understanding of mathematical concepts.

Visual representation plays extremely important roles in mathematics education. These important roles can be categorized as follows:

a) Thinking through what is represented (as a method of thinking)

b) Recording what was thought through representations (as a method of recording)

c) An important method for communication.

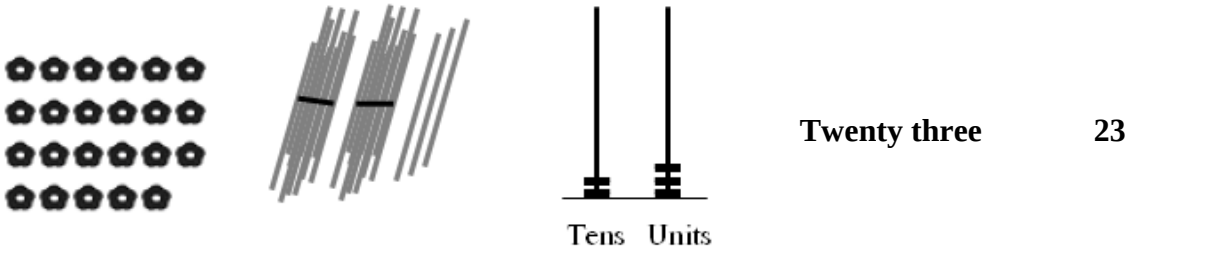
When the American cognitive psychologist Bruner focused on the cognition of children, as well as representative thinking, he pointed out that it is possible to divide representation into the following three classifications, which describe the sequential development stages of representation:

(E) Enactive representation

(I) Iconic representation

(S) Symbolic representation

His system is referred to as the "EIS principle", based on an abbreviation of the three stages of representation. In this talk, the visual representations are in the category of iconic representation.



Visual representation, as both the product and the process of creation, interpretation and reflection upon pictures and images, is gaining increased visibility in mathematics and mathematics education. When students use visual representation as a tool of thinking, the student performance will be observable during the process of constructing mathematical concepts. The figure below illustrates how the visual representation supports formative assessment.

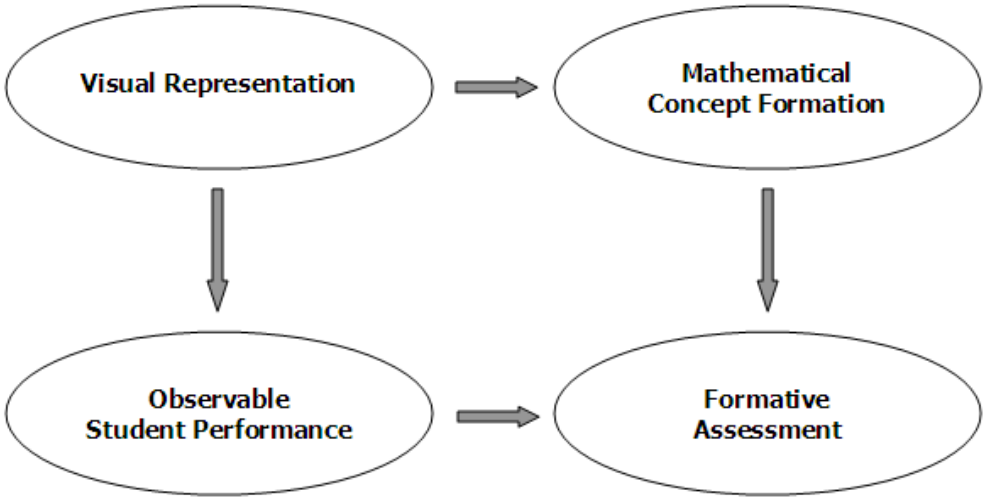
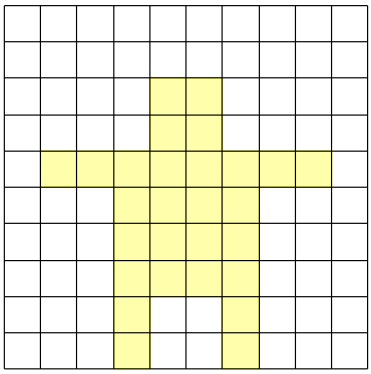


Figure 1. *Visual representation supports formative assessment*

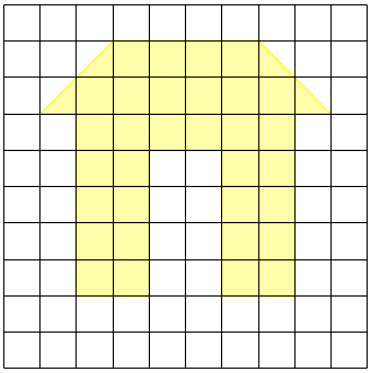
Example 3 (Grade 5). Use Figure 2. Student A shades any number of squares on one of the 10×10 square-grids. Student B answers the questions below the square-grid. Student A checks the answers given by Student B. Each student shades the square-grid twice and answers the questions twice.



No. of squares: _____

Fraction: _____

Percentage: _____



No. of squares: _____

Fraction: _____

Percentage: _____

Figure 2. *Some student liked to draw beautiful polygon shapes*

Some students liked to draw beautiful pictures such as dogs, houses, robots with the polygon shape and asked their peers to answer the above questions. They realized that the shaded region was not necessary a set of small squares but also any polygon that its area can be found easily. To do this activity most students feel that they were free to raise the questions, and knowledge of mathematics constructed was from their figures not from teacher.

4. FORMATIVE ASSESSMENT WITH VISUAL REPRESENTATIONS IN LESSON STUDY

Formative assessment has the purposes of advancing students' learning and informing teachers' instructional decisions. It may occur throughout a teaching cycle using open ended tasks and mathematical investigations.

Example 3 (Grade 5). Make a train with the length of 16 cm.

This is an open-ended task that requires students to make a systematic list of all abilities. The restricted condition is given but the sum of two numbers is unknown.

N. of red wagons	8	6	4	2	0
N. of blue wagons	0	1	2	3	4
Total of wagons	8	7	6	5	4

There are 5 answers to this task. Students know how to analyze a natural number into sum of two natural numbers with a specific restricted condition.

$16 = 8 \times 2 + 0 \times 4$ $16 = 4 \times 2 + 2 \times 4$ $16 = 0 \times 2 + 4 \times 4$

$16 = 6 \times 2 + 1 \times 4$ $16 = 2 \times 2 + 3 \times 4$

From the table students will see that a train with the length of 16 cm including 6 wagons has 4 red wagons and 2 blue wagons. The restricted condition of this problem is:

Sum: N. of red wagons + N. of blue wagons = 6.

Restricted condition: The length of the train is 16 cm.

If all wagons are red then the length of the train decreases: $16 - 6 \times 2 = 4$, then the number of blue wagons: $(16 - 6 \times 2) \div 2 = 2$.

Students practice this procedure to consolidate what they have learnt. The most important fact that the students need to realize is the difference 2 cm between one blue wagon and one red wagon.

Classroom assessment in teaching and learning, formative assessment is based on constructivist and sociocultural theories of learning. In such assessments, students work on complex extended tasks where they make "arguments which describe conjectures, strategies, and justifications" (Romberg, 1993, p. 109) investigated or used during the task.

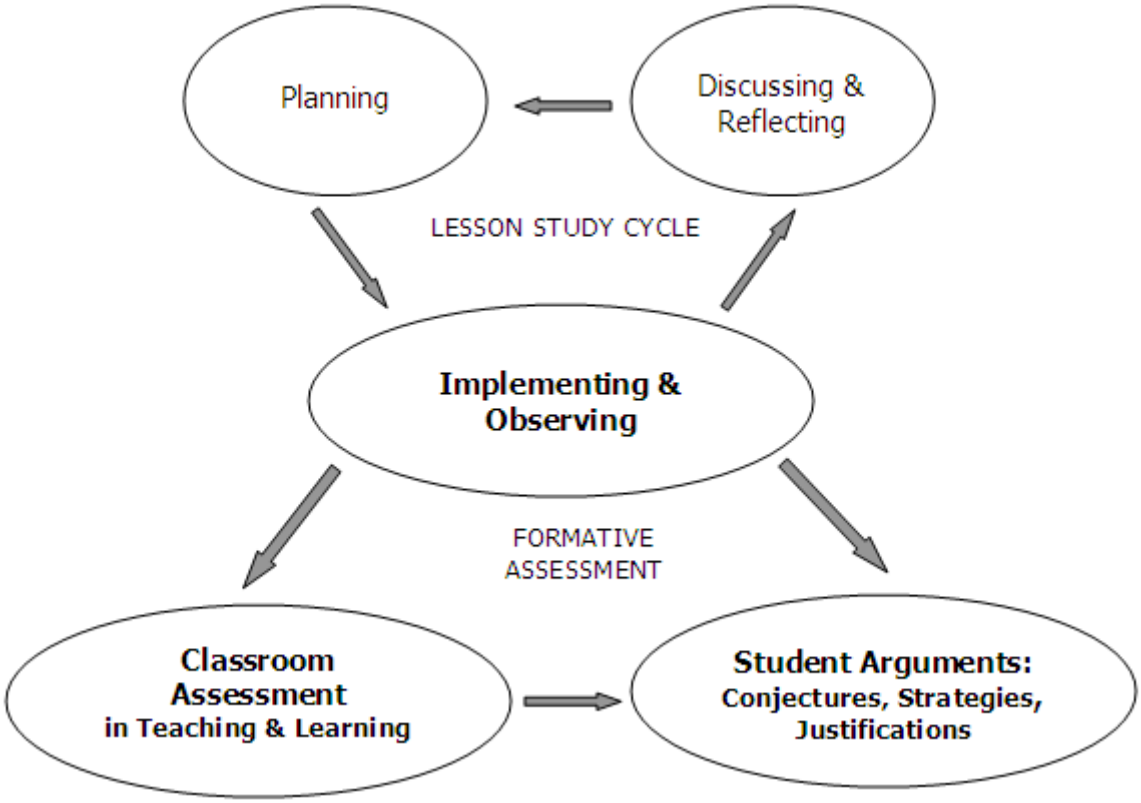


Figure 3. *The connection between formative assessment and the lesson study cycle*

Different forms of assessment are also useful for the different purposes of assessment, and the different stages of learning a new topic. Assessment tasks in themselves should be worthwhile learning activities for students. Formative assessment collects and interprets evidence of

student learning. To implement a good lesson study requires worthwhile activities that serve as rich formative assessment tasks. Rich assessment task encourages a mathematical investigation with open-ended tasks.

In the formative assessment with open-ended tasks, the teacher has chance to observe and makes a judgment about the student's demonstration of a skill or competency in creating a product, constructing a response, or making a presentation. In these tasks, students are required to perform, create, construct, produce, or do something; and to explain, justify and defend.

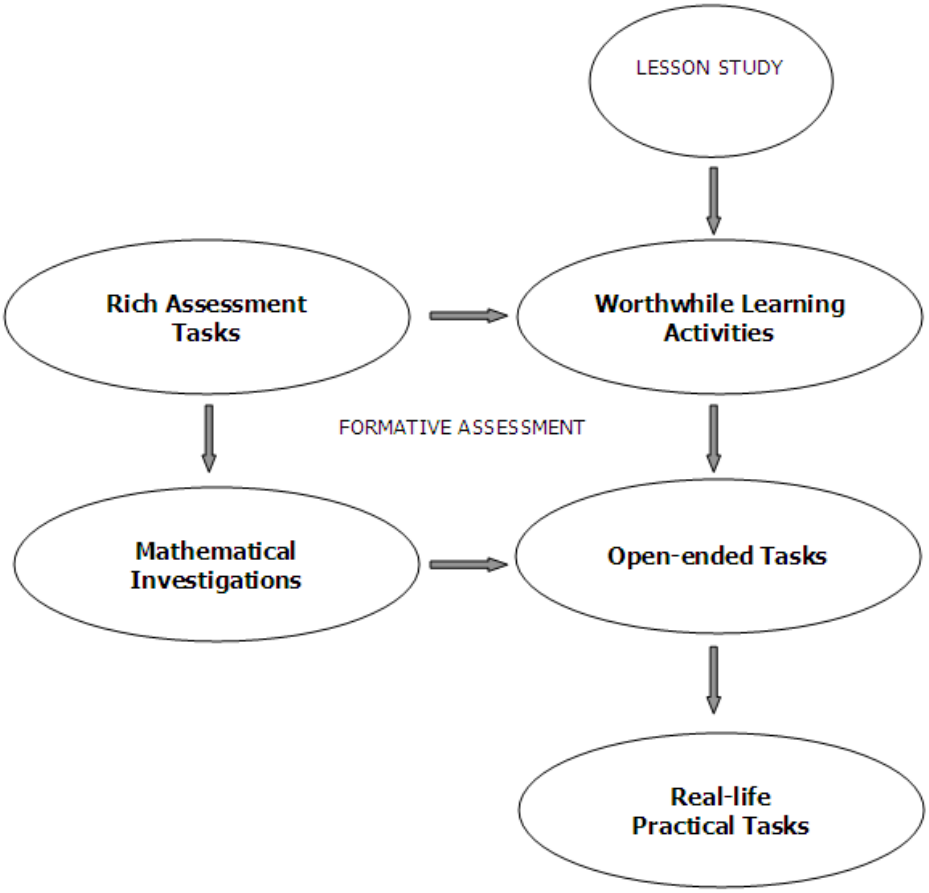
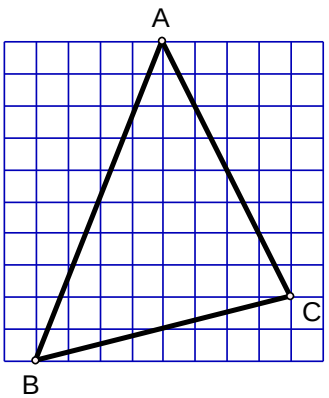


Figure 4. *Rich formative assessment tasks in lesson study*

Example 4 (Grade 7). Students are given a triangle drawn on A2 grid paper. Grid 5cm×5cm.



Determine the midpoint of each leg. Draw three medians of the triangle, identify their lengths.

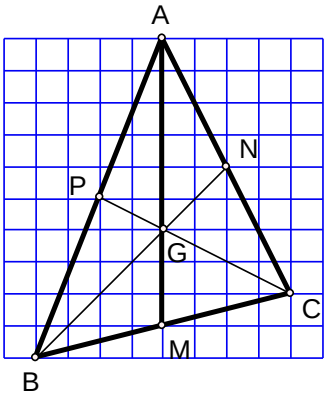
Q1. What is the relation of three medians?

Q2. Can you find any ratio of the lengths of segments that determined by the medians?

Property:

In a triangle three medians are convergent at centroid, and the length from centroid to a vertex is $\frac{2}{3}$ of the median passing through that vertex.

By having students engage in determining the midpoint of each edge, drawing three medians and identifying the lengths of segments that determined by the medians. Three medians are convergent at point *G*.



$AG = 2GM$; $BG = 2GN$

$CG = 2GP$

5. TEACHING APPROACH THAT ENHANCES FORMATIVE ASSESSMENT IN MATHEMATICS

It is difficult to define what is the appropriate teaching approaches (problem solving, investigation, inquiring, discovering) in our countries to enhance communication. We can define some characteristics or principles of a teaching approach supporting formative assessment.

Characteristics of teaching model that enhances formative assessment

- Challenging problems;
- Opportunities for students explain their own ideas;
- The problematic task that students can get solutions at different levels (intuitive, prove, difficulty);
- All students can engage in the lesson;
- Long term planning; teacher should know the history of student's learning; the progress of student' thinking;
- Exploring;
- Inquiry-driven instruction and not clearly structured.
- Investigative;
- Interactive;
- Students write their own understanding on the note books.

Teaching Approach

- Stage 1: Teacher introduces an accessible mathematical task that students can get solutions at different levels.
- Stage 2: Students cooperatively work in small group to explore, get their solutions;

- Stage 3: Students present the solutions to the whole class for discussion; explanation, clarification.
- Stage 4: Reflecting, conclusion of the teacher, students write their own understanding on their note books;

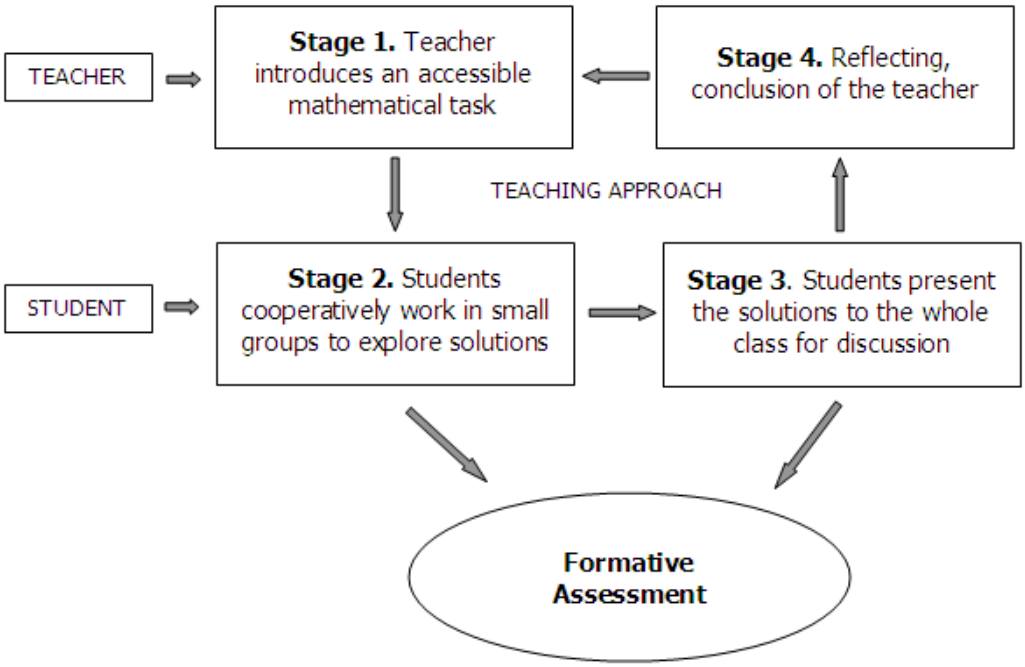


Figure 5. *Teaching approach in lesson study that promotes formative assessment*

6. CONCLUSION

The use of innovation to teaching and learning mathematics in the classroom must be implemented to engage students in meaningful mathematical tasks that require higher order thinking. The innovation provides all students access to a broad range of mathematical ideas. Lesson study guides teachers to focus their discussions on getting the effective innovation through the cycle. In the implementing stage of lesson study cycle, we can use open-ended tasks as development tasks. By discussing and sharing new ideas on implementing a specific subject matter, and then observing what happens in actual classroom, teacher improves their teaching and enhances the students learning based on the evidence of informal formative assessment. We can apply lesson study to many topics in the curriculum. Because of its role in formative assessment, lesson study as a means to innovation actively affected to teaching and learning mathematics in the school.

The innovation as a product of the lesson study helps students have better and meaningful understanding of difficult mathematical concepts. Students were able to discuss and interact freely with their pairs/groups while answering open-ended questions relevant to them. The students communicated friendly their mathematical thinking while they are engaged in the mathematics activities. With visual representations or hand-on activities, students always have something to share with their friends about problems involving with mathematical thinking. More contextual real-life problems and open-ended task that is challenging will make a more rich mathematics class.

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