INCORPORATING MATHEMATICAL THINKING IN ADDITION AND SUBTRACTION OF FRACTION: REAL ISSUES AND CHALLENGES

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Since the introduction of the new primary mathematics curriculum in Brunei starting January2006, more teachers of elementary schools are seeking a suitable way of incorporating mathematical thinking in each mathematics lesson that they teach. For this particular purpose, teachers were introduced to lesson study and it is hoped that with the guidance and support given by the team involved, teachers would build their confidence and make mathematical thinking a regular feature of their lessons. There are still many problems that the teachers face such as students not used to explaining their thoughts in class and some insisting on using certain procedures that they had learned before, without being able to explain how and why the procedure works. This paper will relate a classroom case and look at the real issues and challenges that Bruneian teachers faced in incorporating mathematical thinking when teaching of the topic on addition and subtraction of fractions.

Introduction

Mathematical thinking is the mathematical mode of thought that we use to solve any problem in our daily life including at schools (Khalid, 2006). It can be defined as applying mathematical techniques, concepts and processes, either explicitly or implicitly, in the solution of problems (Khalid, 2006). It is, according to Katagiri (2006), the most important ability that mathematics courses need to instill because it makes students able to think and make independent judgement. He also said that mathematical thinking allows for an understanding of the necessity of using knowledge and skills as well as learning how to learn by oneself, and the attainment of the abilities required for independent learning. Stacey (2006) reiterated this fact by saying that mathematical thinking is important because it is an important goal of schooling; it is important for teaching mathematics; and it is an important way of learning mathematics. In fact, the framework used by PISA shows that mathematical literacy involves many components of mathematical thinking, including reasoning, modelling and making connections between ideas. It is therefore imperative that mathematical thinking be stressed in any school curriculum and this is reflected in the new curriculum for primary mathematics of Brunei Darussalam, which was put into implementation from early 2006 (Khalid, 2006).

Children are encouraged to use thinking skills and problem solving strategies during mathematics lessons and not just learn mathematical skills and concepts from listening to the teachers. It is feared that if mathematical thinking is not emphasized,

our children would end up learning mathematics by rote memorization, without understanding and without the ability to think intelligently.

More on Mathematical Thinking

The new Bruneian "Mathematics Syllabus for Lower and Upper Primary Schools" (CDD, 2006a; 2006b) considers mathematical thinking as among the processes, skills and values that need to be developed through the teaching and learning of mathematical content. This bears similarity to how Professor Katagiri (2006) defined mathematical thinking. According to him, mathematical thinking can be divided into three categories:

I. Mathematical Attitudes
II. Mathematical Thinking Related to Mathematical Methods
III. Mathematical Thinking Related to Mathematical Contents

The first category is considered as the driving force behind the two latter categories.

"Mathematical attitudes" is a very important affective factor in determining students' behavior in mathematical thinking and problem solving because students' attempts in mathematical thinking depend on how interested they are in problem solving or the lesson. Students' expectation that mathematics will be useful (which involve beliefs) and their personal attributes such as confidence, persistence and organization are mentioned by Stacey (2006) as some of the skills and abilities required for problem solving. Attitudes and values are also mentioned in the Brunei curriculum document (CDD, 2006a; 2006b).

"Mathematical thinking related to mathematical methods" was listed in detail by Katagiri (2006) as consisting of inductive thinking, analogical thinking, deductive thinking, integrative thinking, developmental thinking, abstract thinking, thinking that simplifies, thinking that generalizes, thinking that specializes, thinking that symbolizes and thinking expressed with numbers, quantifiers and figures. Stacey, (2006) quoting from Mason, Burton and Stacey (1982) defined this category as mathematical process that is made up of:

- specializing – trying special cases, looking at examples
- generalizing – looking for patterns and relationships
- conjecturing – predicting relationships and results
- convincing – finding and communicating reasons why something is true

The resemblance of this category in the Brunei curriculum document (CDD, 2006a, 2006b) would be the processes which include mathematical thinking and communication.

"Mathematical thinking related to mathematical content" include ideas of sets, units, expressions, operations, algorithms, approximation, fundamental properties and formulas. These can be compared to mathematical skills (as well as estimation and mental computation) in the Brunei curriculum or deep mathematical knowledge as stated in Stacey's requirement for problem solving.

In the Bruneian syllabus, mathematical thinking and problem solving are mentioned together. Teachers must encourage children to use thinking skills and problem solving strategies during mathematics lessons (CDD, 2006b, p. 7). Among the sub-processes of the mathematical thinking and problem solving processes that are listed in the syllabus are: guessing and checking, drawing diagrams, making lists, looking for patterns, working backwards, classifying, identifying attributes, sequencing, generalising, verifying, visualising, substituting, re-arranging, putting observation into words, making predictions as well as simplifying the problem and solving part of problems.

The curriculum recommends the use of a variety of representations to facilitate the development of the content knowledge and processes. Active learning is encouraged and the use of different representations is to be implemented according to the age and stages of the pupils. In the early years, concrete materials are supposed to help children develop basic mathematical concepts. As children move on, diagrams, real-world examples, verbal representations, ICT and symbolic representation will help children proceed from the concrete to more abstract ways of thinking. The use of symbolism to shift from process to concept is what Tall (2006) termed as 'procept'.

The Lesson Study Group

Lesson study was recommended by Khalid (2006) as a professional development for teachers to familiarize themselves with incorporating mathematical thinking in their lessons in Brunei. Since many Bruneiean teachers are not very familiar with lesson study, a long-term strategy in the form of research project was developed to introduce lesson study, in order to make it a regular feature in teachers' professional development and training. A team was established to ensure the smooth running of the project. The team comprises of me (as the project leader), Md. Khairul Amilin Hj Tengah as well as Dr. Hjh. Zaitun Hj. Taha from Universiti Brunei Darussalam, Mr. Palanisamy Veloo from the Curriculum Development Department, and Mr Masaki Takahashi from Sultan Saiful Rijal Technical College. We managed to secure a research grant from the university to help us finance this research study which was approved in June. We managed to attract four teachers attached at a particular Secondary School to join our project and we started the training way before the research grant was approved to make sure they have enough time to consolidate the ideas behind lesson study before we start with actual classes. During training, the philosophy and process of lesson study were explained and with the help of the lesson study videos that were received from Japan, the teachers could clearly see what was meant.

We started our actual lesson around mid April and since the research grant was not yet approved at that time, we manage to record the lessons with a DVD camera belonging to one of us, without even a tripod. When we were about to start the actual teaching process, two teachers withdrew. Of the remaining two, only one attended the

meeting regularly to discuss her lesson plans with us. We managed to record the lessons of both teachers teaching different topics. However, for the purpose of this presentation, we will only look at the lessons of one particular teacher.

The Study

The study involved two secondary one (about grade/year 7) classes, 1A and 1E. The two classes comprised 34 and 38 pupils respectively. The main aim of the lesson is to incorporate mathematical thinking into teaching and learning of addition and subtraction of fractions. There are other aims stated in the teacher's lesson plan and they are as follows:

At the end of the lesson, students should be able to:

I. Perform the addition and subtraction of fraction involving:
   - Fractions with like denominators
   - Fractions with unlike denominators
   - Improper fractions and mixed number

II. Solve problems related to addition and subtraction of fraction

Lesson Plan Development

The lesson plan for the purpose of lesson study was written by the teacher after discussion with the team. She was told that since the purpose of the lesson is to incorporate mathematical thinking, she also needs to think of the kinds of questions to ask the student and what to expect from students' responses. She was determined to make students participate during the lesson because communication is necessary for developing mathematical reasoning. The lesson plan can be seen in Appendix A.

For introduction, the problem that she posed for the students was composed to make students interested in the lesson and to stimulate their thinking. This is considered an important part of the lesson because it dealt with mathematical attitude. It is one of the ways to motivate students, as was mentioned in Keller's (1983) ARCS model where students' attention is gained and maintained by innovative posing of problem. The teacher adapted the names of famous international stars and weird long names for this purpose. The teacher also tried to cover the syllabus to include like, unlike and improper fractions in her introductory problem.

Next, the teacher prepared teaching aids in order to help students visualize the problems in a concrete way which can be classified as thinking expressed with figures or manipulatives. Her paper folding activity is also an attempt to make students translate thinking that symbolizes to thinking expressed with figures. The ability of students to translate parts of a round pizza to rectangular parts also involved mathematical thinking because it requires the ability of students to simplify and translate the problem to another equivalent form. She also tried to make children think by generalizing when she asked students to look at patterns (or what happen) when the questions were changed to fractions with larger denominator. At the same time, she planned for students to conjecture about the result when this happens. The

learning processes in conjecturing include defining, exploring and constructing premise/conclusion according to Fou-Lai Lin (2006) or predicting relationships and results according to Stacey (2006).

To further assess students' understanding of the lesson, the teacher prepared different sets problems involving fraction magic-squares. Number puzzles and tricks are excellent for featuring mathematical thinking prominently in lesson (Stacey, 2006). Students were made to work in pairs and each pair was given a different set of magic-square. Working in pairs requires ability such as communication and interpersonal skills. The lesson was planned well and the team was eager to observe the lesson.

Lesson Observation and Comments

Below are the comments on the implementation of the lesson based on the observation data that was collected. I will try to identify the elements of mathematical thinking that are present during the lesson.

The teacher attempted to present the lesson via problem-solving strategy. When the problem was posed in the context described, the element of mathematical attitude (willingness to attempt and attempting to discover mathematical problems in daily life) was present. Students were observed to be very excited and can be heard repeating some of the weird and famous names. The teacher was therefore successful in making students interested in the lesson as well as stimulated their thinking. Later, she pasted the teaching aids in the form of paper pizzas on the white board. Interest, enthusiasm and attitude are important to arouse curiosity and she has done that successfully. She then proceeded by asking the students to give her the mathematical statements of the problems. Students had done fractions before and they can therefore build on experiences that were *met-before* (Tall, 2006). The children responded by giving chorus answers. Here, ideas are compressed into thinkable concepts using language and symbolism (Tall, 2006). At this stage, the teacher had used four of the representations suggested by the curriculum – real-life, diagram, verbal and symbolic.

She then proceeded to use the concrete representation (manipulatives), the paper-folding activity. Therefore the only representation that she did not use was the ICT, which I think would be one of the best representations for better understanding of addition and subtraction of fraction. She could have brought the students to the audiovisual room for ICT representation since the facilities there would allow the use of ICT. However, the use of concrete representation involved some amount of mathematical thinking when students need to transfer other representations to this representation. Since the shape of the paper used is not the same as the one in the diagram, children need idea of units (mathematical content) and focus on the constituent elements and their size and relationship (Katagiri, 2006). Children were encouraged to fold the rectangular papers guided by their teacher to understand how the answers were obtained. In the case of unlike fractions, most students prefer the use

of lowest common multiple (LCM), which is the procedural knowledge that they had acquired before. Students do not really understand the idea of LCM since even for simple LCM of 4 and 8, they needed to use the algorithm that they learned before, and did not use logical thinking at all. The teacher however, did not take this opportunity to explain LCM in terms of finding equivalent fractions. This is a common feature of a mathematics lesson in Brunei, where teachers teach procedures and algorithm without explanation and students learnt them without understanding. So they were able to do mathematics without understanding what the lesson is all about. There are other instances where rules were given and children try to remember them by heart. When they are many rules to remember, they would be confused and make mistakes.

Students were able to generalize and conjecture (mathematical method), when the teacher guide them to look at the patterns and when fractions with large denominators were used. Since folding paper would be out of the question, this is considered as an important part of the lesson. However, it would have been better if she asked one of the children to communicate their reasoning by asking appropriate questions instead of just getting chorus answers. Most of the explanations given by students were not very convincing.

Before the end of the lesson, children were provided with magic-squares where they have to fill-in the empty boxes. Children were observed to communicate with each other and the teacher to discuss the problem. Although clear instructions were given to the students, almost all of them could not complete the problem.

During discussion with the team, the teacher was advised to teach addition and subtraction of fractions separately, because she had to rush through the lesson due to limited time. The teacher responded that her lesson was planned that way because the students had learned about addition and subtraction of fractions before. However, she will separate them into two lessons for the other class that she would teach that week since it seemed that students still need time to understand about fractions. Our comments were taken positively and we could see some improvements when she taught subtraction of fraction to class 1E. Here we could see that by teaching addition and subtraction separately, the phase of the lesson in 1E was just right. Furthermore, these students are supposed to be a weaker group than the previous one. We were not able to observe the teaching of addition of fraction for class 1E because many of the team members had other commitments on that day.

Discussion

So, what are the real issues and challenges related to the incorporation of mathematical thinking in each lesson in Brunei Darussalam? In my last paper (Khalid, 2006), I have mentioned three perceived issues and challenges:

1. The over-emphasis on examination and examination results.
2. Teachers readiness to teach students to think mathematically
3. Changing the expectations of the stake-holders.

I will however add another one and that is lack of students' participation during lesson.

The first issue is still the biggest issue to be addressed. The children involved in this study were children who sat for their "Primary Certificate Examination" the year before. Considering they have gone through fraction with their teachers during primary levels, one would expect a solid understanding of the topic from them. However, looking at the way they tried to solve the magic-square, it seemed like they were learning the topic the first time. Their persistence on using LCM to solve addition and subtraction of unlike fractions proved the case that they remember algorithm and do not actually understand what LCM stands for. This agrees with the findings of Lim (2000) and Clements (2002) that said that Bruneian children have low level of conceptual understanding of mathematical ideas involved and rely on procedural approaches or rote memorization. Tall (2006) reiterated the fact that procedures that are not compressed into thinkable concepts may give short-term success in passing tests, but if those procedures are not given a suitable meaning as thinkable concepts (in this case, procepts), then they may make future learning increasingly difficult.

The need to reform assessment and evaluation of school children becomes more crucial. Assessment should vary and should not solely depend on one sit-down examination to determine students' progression. At the elementary level, more performance-based and authentic assessment should be introduced. Traditional testing methods in mathematics have often provided limited measures of student learning, and equally importantly, have proved to be of limited value for guiding student learning. These methods are often inconsistent with the increasing emphasis being placed on the ability of students to think analytically, to understand and communicate, or to connect different aspects of knowledge in mathematics. I am however pleased to hear that the ministry of education is doing something about this. As a first step, they could at least set the examination questions into those that need mathematical thinking to solve.

The second issue concerning teachers' readiness to teach students to think mathematically is really an important issue. Teachers are not used to teach this way and have not been exposed to this kind of teaching. Although the idea and method was taught to them during teacher training, they tend to go back to the traditional method once they were posted to schools. I guess old habits die hard. In the attempt to expose teachers to mathematical thinking through lesson study, I have not had the opportunity to extend this to other schools, due to lack of time and resources since the grant for the purpose of lesson study was only approved in June. We also need committed teachers who are interested in lesson study to make it a success. We went through a setback when teachers decided to withdraw from the study. I guess the school should introduce a reward system for teachers who participate in lesson study

or other professional development courses. However, I am still hopeful that it will succeed when I hear about the success of lesson study in other countries.

I consider the third and last issue as the most difficult to change. Stake-holders like parents and children should be made to realize that putting too much emphasis on examination results will in the long run lead to the children being disadvantaged. There should be an awareness that the nature of work has changed. Employers nowadays seek workers who are team-players, thinkers and problem solvers. Mathematical thinking provides a solid foundation to produce good problem-solvers. School administrators should not put too much pressure on the teachers to produce good result, because there is a tendency for teachers to take a short-cut to achieve this. Teachers become pre-occupied with preparing students for examinations (Majeed, Aldridge& Fraser, 2001) and students would come to regard mathematical "understanding" being the same as being able to answer examinations questions correctly (Clements, 2002). If we do not change, then the level of scholastic ability of the students will be among the top of the list produced by Katagiri (2006) in the hierarchy of scholastic abilities and mathematical thinking (from lower to higher) reproduced below:

1. The ability to memorize methods of formal calculation and to carry out these calculation
2. The ability to understand the rules of calculation and how to carry out formal calculation
3. The ability to understand the meaning of each operation, to decide which operations to use based on this understanding, and to solve simple problems
4. The ability to form problems by changing conditions or abstracting situations
5. The ability to creatively make problems and solve them

The higher the level, the more important it is to cultivate independent thinking in individuals. To this end, mathematical thinking is becoming even more and more necessary.

Students' reluctance be active participants in class can be corrected through constant encouragement from teachers. There are many reasons for them to be quiet in class, such as being afraid of making mistakes and not being fluent enough in English. However, classroom culture can change if teachers insist and encourage certain behaviors and I am glad to see that more and more of our teachers encouraging students to speak and not being afraid of making mistakes.

Conclusion

Mathematical thinking has been proven to be important and is therefore emphasized in the new Bruneian "Mathematics Syllabus for Lower and Upper Primary Schools". The success for implementing mathematical thinking needs concerted effort from everyone involved. The country needs a well educated workforce with the ability to think and analyze, using varied reasoning and problem-solving skills in an integrated manner for national development. In order to be able to independently solve problems and expand upon problems and solving methods, the ability to use **"**mathematical thinking**"** is considered even more important than knowledge and skill, because it enables to drive the necessary knowledge and skill (Katagiri, 2006).

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APPENDIX A

Lesson Plan of the First Lesson Observed

Name of Teacher: Hajah Nuzailah
Class: 1A (Year/Grade 7)
Topic: Addition and Subtraction of Fraction.

Learning Objective: So that children are able to think mathematically how to solve problems related to addition and subtraction of fractions (like and unlike fractions).

Other Objectives: 1) For students to be interested in the lesson (have the right attitude to involve in mathematical thinking)
2) For students to think mathematically in integrating what they know and relate it with what they are currently learning and to make sense of the lesson using the manipulatives provided.

Introduction: Pizza problem

These students ordered some pizzas from the pizza vendor and ate them according to the following proportion:

Name	Amount of pizza
1. John Beckham	1/3
2. Ramziah Khuzaifah	1/3
3. Shawn Wayne	1/5
4. Agus Muslimah Qawiyah	3/8
5. Hendrick Schumacher	1½

Questions Posed	**Corresponding thinking**
1. What is the amount of pizza John and Ramziah have together?	Students are supposed to add like fractions
2. If John and Ramziah gave up 1/3 of their total share, what is the amount they have left?	Students are supposed to get the sum from question1 and subtract ¼ from it. Subtract like fractions
3. What is the amount that both Shawn and Agus have together?	Adding unlike fraction
4. What is the amount if Shawn and Agus give up ¼ of their share to a friend?	Getting the sum from question 3 and subtract ¼ from it
5. What is the amount that Shawn and Hendrick have together?	Adding unlike fractions involving improper fraction
6. Shawn and Hendrick gave up 2/3 of their share. How much do they have left?	Getting the sum from Question 5 and subtract 2/3 from it
7. How many pizzas did they order altogether?	Getting the sum of all the fraction of pizzas And rounding them

Activity: Paper Folding

Students are given blank papers, and are encouraged to translate the questions using he rectangular paper to get answers (by folding and shading). Students were asked to look at patterns and as follow-ups, the problems are extended to larger denominators like $13/20 + 4/20, 2/15 + 4/12, 1\frac{7}{8} - 1\frac{1}{3}$

Student Evaluation: Magic Square with fractions

Students were supposed to work in pairs and fill up the empty boxes with fractions that will make each row and column equal to 1. They are not supposed to use the same fraction more than once.

Lesson Plan of Second Lesson Observed

Name of Teacher: Hajah Nuzailah
Class: 1E (Year/Grade 7)
Topic: Subtraction of Fraction.

Learning Objective: So that children are able to think mathematically how to solve problems related to subtraction of fractions (like and unlike fractions).

Other Objectives: 1) For students to be interested in the lesson (have the right attitude to involve in mathematical thinking)
2) For students to think mathematically in integrating what they know and relate it with what they are currently learning and to make sense of the lesson using the manipulatives provided.

Introduction: Is there enough Pizza?

Jamil bought one large pizza and plan to share the pizza with his friend. He divided the pizza as follows:

Name	Amount of pizza
1. Jason	¼
2. Jill	1/8
3. Jamilah	3/8
5. Johari	2/5

Questions Posed	**Corresponding thinking**
8. If Jason is the first to eat the pizza, how much is left for the rest	Students are supposed to subtract a quarter from 1 to get three quarters.
9. If Jamil gave 1/8 of the remainder to Jill, what fraction is left?	Students are supposed to subtract 1/8 from three quarters
10. During break time, Jamilah ate 3/8 of the remaining pizza. How much pizza	Subtracting like fraction

is there left?	
11. Is there enough pizza for Johari? Is his share bigger or smaller than Jamilah?	To test students' understanding on order of fraction (which is bigger and smaller
12. What happen to Jamil's share? Is it getting more or less?	To make student think what happen during subtraction

Activity (Paper folding)

Students are given blank papers, and are encouraged to translate the questions using he rectangular paper to get answers (by folding and shading). Students were asked to look at patterns and as follow-ups, the problems are extended to larger denominators like $13/22 - 4/22$, $4/15 - 2/12$, $1\frac{7}{8} - 1\frac{1}{3}$. Although there was no question involving mixed number or improper fractions above, children were asked one question involving these fractions during the activity.

Student Evaluation: Worksheet on subtraction of fraction

The questions on this worksheet were the normal exercise book problems that we often see.

Appendix B (Video Description)

Title: Addition and Subtraction of Fraction

Teacher: Hajah Nuzailah Haji Nali

Class: Form 1A and 1E (Grade 7)

School: Sekolah Menengah Masin

Date: 1st lesson – 14th April, 2007 for class 1A
2nd lesson – 24th April, 2007 for class 1E

Team member: Dr Madihah Khalid (UBD), Mohd. Khairul Amilin Haji Tengah (UBD), Dr Zaitun Hj Taha (UBD), Mr Masaaki Takahashi (MTSSR), Mr. Palanisamy Veloo (CDD)

Introduction
The first lesson was on the topic of addition and subtraction of fractions. The teacher planned the lesson very well, laboriously prepared the teaching aids, determined fully the procedures of her teaching and prepared interesting activities. Everything was written in her lesson plan as shown in Appendix A. Since the aim of the lesson was to incorporate mathematical thinking into each lesson, she has also prepared to ask appropriate questions to students directly to elicit students reasoning.

In the introduction of the lesson, the teacher tried to motivate the students by telling a story so that children would be interested to learn and be involved in class. This is one aspect of mathematical thinking, (the mathematical attitude) that the teacher is trying to address. The names of the characters in the story were adapted from the names of famous people. She probes and asks students questions but usually gets chorus answer. She could have improved her questioning techniques if she set some rules like whoever wants to speak in class should raise their hands-up and she would in turn call out the children. From our discussion, we think that she should have asked the students why "when it comes to addition and subtraction, the denominator need to be the same".

Activity (Paper folding)
Paper folding was used to help students translate numbers and symbols to the concrete form for better understanding. Transferring round pizzas into rectangular papers also needs imagination and is also another aspect of mathematical thinking. It is also here that the teacher should stress the importance of equal parts, to allow these parts to be added or subtracted together. Maybe, ask one of the students to explain this.

Some students are not really interested in folding papers since they were already taught the addition/subtraction algorithm. When faced with addition or subtraction of unlike fractions, they would always suggest finding the "LCM" or "least common multiple". I found that children don't really know what LCM really is. They just know

how to find them. Even when asked “What is the LCM of 2 and 4, they insist on performing the division” (algorithm).

Children are still not communicative enough and this is where a teacher’s skill in probing would be handy. In my opinion, students in Brunei are still not participative enough in the class and this lack of communication and the inability for them to explain their thoughts make it a challenge for teachers in Brunei in reading their thoughts (mathematical thinking).

Assessing students’ understanding
Teachers are supposed to encourage students’ self evaluation by asking right questions. However, only a few pupils could explain well. Therefore, to further assess their understanding, students were asked to work in pair to work out the answers to the magic square with fractions. Different pairs of students were given different number combination. An example of one of the combination given is as follows:

	$\frac{10}{20}$	$\frac{3}{9}$
$\frac{4}{12}$		
		$\frac{6}{16}$

Children were found to still struggle to complete this simple exercise and they could not finish it in class because time was up. They were however encouraged to complete it at home and bring to the next lesson. During discussion with the teacher, she was advised to just concentrate on addition in one lesson and subtraction in another lesson, because one hour is too short for both.

The Second Lesson

The second lesson that the team observed was a lesson subtraction of fraction with class 1E. These pupils had a lesson on addition of fraction five days before and the team could not observe the teacher.

Evaluation of the lesson
Again, to make the lesson interesting, names of students in the class were used. It aroused students’ interest and stimulated their mathematical thinking. This lesson was better executed than the previous one and children enjoy it because they have enough time to think and do the activity with their teacher. However, there is still the tendency for pupils to solve problems using LCM. Although the teacher tried to make them think in terms of equivalent fractions, they still insist on solving it using LCM. I guess old habit dies hard.