PROMOTING MATHEMATICAL COMMUNICATION IN THE CLASSROOM THROUGH SOLVING OPEN-ENDED PROBLEMS

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This paper describes how by improving a lesson involving open-ended problems that they had collaboratively developed and implemented in 2007, two Filipino teachers who are members of a lesson study group created more opportunities for pupils to think and communicate mathematically. By deliberately and systematically planning the kinds of questions that they ask and the manner that they ask them and anticipating the possible pupil responses, they experienced drawing out important mathematical ideas from the pupils themselves. The pupils also experienced solving open-ended problems.

SOME BACKGROUND INFORMATION

Learning from Previous Work

Last year, the teachers in the lesson study group while planning for a lesson on solving word problems involving subtraction with regrouping, discovered what an open-ended problem is and what solving it means (Ulep 2007). This was part of their important achievements in the lesson study. This year, the group viewed the videotape of the teachers' last year's actual implementation of this lesson in their own classes and realized that their questions and how they asked them needed to be improved. They also realized that the lesson introduced so many changes in teaching the topic that there was not enough time to process the pupils' responses to the open-ended problems. Despite this, they saw indications that the pupils could solve such problems which at the start, they thought the pupils would not be able to do since it was something that they had not done before. So this year, the group decided to provide more time for the new set of pupils to work on the problems, something that the pupils would do for the first time. They would give the pupils more opportunities to explore various methods and answers, interact with others, present their work in class, and process the different works of pupils by evaluating and relating them to bring out significant mathematical ideas. As such, these would give pupils greater opportunities to think and communicate mathematically.

Planning a Lesson that Promotes Mathematical Thinking and Communication

The group recognized the important role that both the mathematical task and questions that the teacher asks play in eliciting mathematical thinking and communication in class. The teachers took the initiative to propose tasks such as the one on the motivation. The task intended to enable every pupil to contribute something to the discussion because it was open. (See the lesson plan.) This was an activity that the teachers rarely do. The other tasks were problems that were the main components of the lesson and these were new to the pupils' experiences. These were taken from the previous year's lesson. One teacher rewrote the lesson plan based on his understanding of the discussions on it during the four planning meetings. The other teacher suggested that worksheets containing the number sentences be given to the pupils to avoid pupils' mistakes caused by misalignment of digits.

There were two problems. Both involved finding the missing digits of a subtraction sentence. Both could be solved in different ways. Both could be considered open-ended in the sense that the methods in finding the answer in both were open. In Problem 1 there was only one definite answer whereas in Problem 2, even the answers were also open (Ohtani 2007, p.9). Following are the two problems:

Problem 1: Find the missing digits of the subtraction sentence below:

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    76   4
 - 5   88
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   43   86
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Problem 2: Find the missing digits of the subtraction sentence below:

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    6   5   4
 -  18   9
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       37   8
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Likewise, during the planning, the important mathematical ideas were carefully identified so that it was clear to the teachers what the lesson aimed at. It was emphasized that these ideas should come from the pupils and their generation should be scaffolded through the teacher's questions. To draw out these ideas from the pupils, the teachers had to formulate appropriate questions. As such, the researcher always prompted the teachers with "What question will you ask so that the pupils will give the answer that you want?" In the process, the teachers learned to ask questions that were more open and that required the pupils to think and give reasons for their responses.

ELICITING MATHEMATICAL COMMUNICATION IN CLASS

Only one teacher could teach the lesson to a new set of grade 4 pupils because the other one had been assigned to teach in grade 5 but he still continued to participate actively in the lesson study meetings. When the lesson plan was finally implemented, all throughout the different parts of the lesson the teacher acted on the principle that the pupils should contribute to the development of mathematical ideas in the classroom and this required communicating mathematically. Following are specific ways by which the teacher created opportunities for the pupils to engage in mathematical communication and these instances were captured in the video.

Asking Pupils to Observe and Describe the Characteristics of Number Sentences Involved in a Problem

In the first problem, the teacher repeatedly required the pupils to observe the characteristics of the number sentence that they were working on. This was to show that it was not only important to solve the problem. It was equally important to observe the characteristics of the number sentence that it involved since these could later explain why the answer to the problem was such. It was also important to be precise and complete in one's descriptions. So in the first problem, the teacher did not only ask the pupils how many digits were missing. Through his questions, they were able to describe that there were five missing digits and there was one missing digit in each column of the number sentence. They were also able to specify in which place values there were missing digits. Based on the logical sequence of questions that he raised, the teacher likewise was able to elicit from the pupils the characterizations of the number sentence in the second problem. These were namely, that it had six missing digits, each of its columns had one missing digit except the tens column which had two.

Articulating the characteristics of the number sentences used in the problems could later enable the pupils to formulate new problems. This they could do by modifying the characteristics of the original number sentences. This could then potentially generate more mathematical communication and new mathematical ideas.

Requiring Pupils to Explain their Answer

The teacher required pupils who presented their work in class to explain how they got their answer. However, what all these pupils did was to simply tell the procedure that they used instead of the process of thinking that they went through to obtain the answer. They were unable to make conceptual explanations of what they did.

Making Pupils Evaluate Their and Others' Work

Each time the pupils had already solved a problem, the teacher would ask "How do you know if your answer is correct?" He wanted them to always verify the correctness of their work. He also asked the pupils who were on their seats to compare their works with those posted on the board, examine what others had done and what they had done, and judge which ones were correct. This was a good way of dealing with the pupils' incorrect responses. However, he did not ask why the responses that were identified as incorrect were so.

In the video, a boy whom the teacher called to give comments on the work posted on the board said in Filipino, the native language, that the first work on Problem 1 was incorrect. In another instance, this same boy pointed out to another boy seated in front of him why the latter boy's answer was incorrect. The boy code-switched from Filipino which he used to give comments, to English which he used to tell the other boy why the latter's answer was incorrect. This language practice is common in mathematics classes in the Philippines (Ulep 2004).

Understanding what Pupils Think and Making Oneself Understood by the Pupils

Only three of the six works on the first problem presented on the board showed the correct answer. The teacher wanted the pupils to realize that all the three works were the same since all their respective digits were the same. And so there was only one correct answer. But when he asked how many correct answers there were, the class had different responses. Those who answered three must have understood the question as “How many of those works are correct?” Those who answered one must have interpreted the question as the teacher desired. Realizing that he was not making himself clear, the teacher asked the pupils to look at the digits of the correct works. This was a very important act of the teacher. He did not tell the pupils that they were incorrect. Rather he made them focus on the digits of the works that they had collectively identified as correct in an attempt to make them see that these were the same and so there was only one correct answer. However, when nobody still could give the desired answer, he already said that there was one.

Building on Pupils’ Initial Responses

Initially, the answers to Problem 2 that the pupils posted on the board did not yet include all the possible answers. They also responded differently to the teacher’s question “How many possible answers are there?” So what the teacher did was to list all the digits that the pupils had used so far to replace the blank in the tens place in the minuend. Then he asked if there were other digits that could be used. Through this, they were able to discover the ones that were not considered yet. This act of the teacher was again very important. He did not tell them the other answers but made it possible for them to discover these by what he did. This act was based on his own decision which he made in class in response to how the events in the classroom unfolded. And this was very good. In effect, he asked them to draw out from the answers presented on the board, all the possible answers. This strategy is an example of explaining inductively (Isoda 2007).

The class was able to consider the digits from 1 up to 9 only. However, instead of perhaps asking a question to make the pupils think of using the digit 0, he already asked them directly if this digit could also be used.

Enabling Pupils to Make Connections

After the pupils learned that Problem 1 only had one correct answer, the teacher asked why this was so. He wanted to make them discover that there was a relationship between this fact and the fact that there was only one missing digit per column in the number sentence used in Problem 1. However, it seemed that they could not make this connection. Apparently, they could not relate that since in each column, two digits were given and only one digit was missing, then only one digit could be used to fill the blank. This is because subtraction is a binary operation. During the lesson planning this reason was discussed. However, the teacher did not explicitly take this up in class.

Similarly, after the pupils had found the ten possible answers to Problem 2, the teacher asked why there were ten such answers. To guide them to discover why, he made them compare the characteristics of the number sentence in Problem 2 with those of Problem 1 which had only one correct answer. He wanted them to realize how they were the same and how they were different, that is, how they were related. The pupils were able to identify the relevant difference which was, that the tens column of the number sentence in Problem 2 had two missing digits. But again, neither the pupils nor the teacher explicitly reasoned out that since only one digit was given on this column and subtraction is a binary operation, then any of the ten digits could be substituted for the blank in the minuend or the difference. Implicitly, the teacher made them work on the argument that if the characteristics of the number sentence were like these, then the number of possible answers to the problem to which it is associated is such. This is reasoning dialectically (Isoda 2007). In other words, by asking the pupils to justify their answer or to pursue the reasons why they got the answer that they did, the teacher probed into the pupils' understanding by engaging them in mathematical communication.

To summarize, by making pupils analyse relationships made possible by solving open-ended problems, the teacher, mainly through the questions that he raised enabled the pupils to be very much a part of the process of coming up with significant mathematical ideas.

MAKING IMPROVEMENTS FOR BETTER MATHEMATICAL COMMUNICATION

There were certain things that the teacher could have done to make the lesson, the first of its kind which the pupils encountered, more effective relative to mathematical communication.

Using Group work

In the lesson plan, the teacher was supposed to make the pupils work in pairs but this was not carried out in the actual lesson. If this was done, there could have been more discussions as pupils would more informally convince their partners about the correctness of their method and answer. Not everyone could have the opportunity to present his/her work to the whole class. But certainly everyone could have the opportunity to discuss with one's seatmate.

Being Clear and Consistent in the Use of Terms

Answer as supposed to be used in this lesson refers to the set of digits that are used to fill the appropriate blanks in the number sentence. So if one digit is incorrect, the answer is incorrect. The digits that were used to fill the tens place in the minuend or difference of the number sentence in the second problem were not the answers themselves. The correct answers consisted of all the digits properly placed in the blanks that made the subtraction sentence a true or correct sentence. Apparently, the pupils did not recognize the relationship between the number of correct answers and the number of possible digits that could be substituted to the tens place in the minuend or difference. This, the teacher should have explained.

The teacher sometimes used the word “number” for digit and accepted pupils’ responses when they interchangeably used these words which might have caused confusion. During the lesson planning, care in using these words was emphasized. And this might explain why at times though the pupils inappropriately used “number” for “digit” in responding to the teacher’s questions, the teacher used “digit” when he emphasized the pupils’ response.

Examining how Pupils Might Interpret the Teacher’s Questions

Pupils sometimes responded incorrectly to the teacher’s question because they interpreted it differently from what the teacher intended. This caused confusion and wasted time. In the planning session, the lesson study group should evaluate how the questions that are raised might be interpreted by the pupils.

Making Explicit Important Mathematical Relationships

For a column in a subtraction sentence with missing digits, if two digits are given, then subtraction or addition can be applied to get their difference or sum, respectively. If only one digit is given, then any digit can be used to serve as the other digit so that these same operations can be applied. These are the underlying ideas that explain why there are ten different correct answers to Problem 2 and only one to Problem 1, given that there is only one missing digit in every column of the number sentences in the two problems except in the tens column of the number sentence in Problem 2 which has two missing digits. The teacher should have made these explicit during the class discussion on the number of correct answers to the problems.

Listening Carefully and Asking Questions

Both the teacher and the pupils should learn to listen carefully to each other. For instance, if the teacher did, perhaps he would have realized that when he asked the pupils to explain their work, they did not really explain how they got their answer but simply described the procedure that they did. And then he could have guided them to really explain since during the lesson planning, he himself had done it. In another instance, the teacher repeatedly asked the question “How many correct answers are there?” but the pupils also repeatedly incorrectly responded. If the pupils listened carefully and tried their best to understand him, then maybe they could have understood him and more quickly at that. Perhaps the pupils could also have asked him questions to clarify their understanding. To take this a bit further, an important practice that should be a part of the classroom culture is not only for the pupils to give comments or ask questions to the teacher but also to other pupils particularly to those who present their work in class, without the teacher prompting them to do so.

CONCLUSION

“After all, the pupils could do it!” This was the comment that the teacher who implemented the lesson gave right after the lesson ended. He was amazed that the pupils could be successful at solving open-ended problems. And it could also be added, “After all, the teacher could do it!” He could teach a lesson that involves solving open-ended problems. In a country where teaching and learning mathematics can be described as predominantly not learner-centred yet, what had happened to this class presents an evidence that through lesson study, mathematical thinking and communication can be promoted through open-ended problem solving.

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