FROM SHARING EQUALLY TO FRACTIONS

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This paper addresses the current situation of mathematics education in Chile and a strategy developed by an University and the Ministry of Education to improve learning in the first four years of the primary school. A comparison is made between the 2006 version of this strategy and Lesson Study, as a whole-school research model. It concludes with a description of a didactic unit for the fourth year of primary school. This is an introductory unit to the study of fractions. A videotape of a lesson from this unit is analyzed.

Primary School and Mathematics Education in Chile.

The Chilean educational system has changed substantively since the 1990's[1]. The national budget has increased significantly, as well as the wages of the teachers, the resources for making learning accessible to more students and the measures of social support to students. School infrastructure has improved, the school working time has been extended and the curriculum has been modernized.

Nevertheless, the transformation of the pedagogical practices has been insufficient with respect to what was expected from the curricular reform. There have been advances in the adoption of more active working strategies and in the incorporation of familiar contexts for students, but it has been observed that these activities are not clearly oriented towards specific learnings, the use of time is barely effective and the classes are weakly structured and planned. These limitations are related to the fact that teachers have to spend 75% of their working time in the classroom.

At the end of the fourth year of primary education, all students in the country take a test in language, mathematics and science. The results of this test have not improved significantly in the last (how many?) years, and a large gap remains between the performance of children of more underprivileged sectors and those that have greater economic and socio-cultural resources.

To correct this inequity, it was deemed necessary to improve the professional development of teachers of the first primary cycle (four years), helping them to

[1] The information outlined here is taken from: Orientaciones para el Nivel de Educación Básica 2004 - 2005, official document of the Ministry of Education.

understand and to implement the new curriculum in mathematics and language; these areas are considered essential to support the rest of school learning. In this context, the Ministry of Education and the University of Santiago de Chile have developed a strategy to support schools in the mathematics curriculum implementation. This strategy aims to improve the educational practices workshops at each school for first cycle teachers, along with support and feedback to the educational activity in the classroom (Gálvez, 2005).

The strategy was implemented in 20 schools in 2003 and expanded to 224 schools in 2004 and 2005. Since 2006 it has been redesigned as LEM communal workshops in mathematics. In this modality, each workshop brings together teachers from two to five schools belonging to the same commune (district), with the purpose of widening coverage to 650 schools, and it will be certified as a training activity, in order to ensure the regular attendance of the teachers. However, there is a risk of weakening the generation of institutional conditions in each school, for the installation and permanence of the changes achieved in teacher's practices.

Lesson Study and Lem Communal Workshops of Maths.

A parallel between Lesson Study (LS) in its whole-school research model version and the Strategy to Support Schools in the Mathematics Curriculum Implementation developed in Chile, in its LEM Communal Workshops of Mathematics version (LCW) is presented in the following table.

According to Yoshida (2005), the steps that encompass a lesson study cycle are:	According to the Terms of Reference elaborated by the Ministry of Education of Chile (2005) LEM Communal Workshops (LCW) are characterized by:
The process begins with defining a broad, school-wide research theme.	The process arises as an initiative of the Ministry of Education to improve teacher training in order to implement the new curriculum in the first cycle of primary education (four years).
Teachers form lesson planning teams and select a lesson study goal.	All the teachers of first cycle from two to five schools of a commune register in a Communal Workshop in which they will work during a year in mathematics and the following one in language, or vice-versa.
The team invites an outside expert to support them.	Ministry and Universities collaborate to produce written and audio-visual materials and to perform assistance activities for the whole process of teaching organization in each school, through a consulting teacher, enabled by the

	Ministry and Universities specialists in charge of the development of the Strategy.
The team selects a unit, and within that unit, selects a lesson topic. Members of the team write a lesson plan based upon research of the topic, instructional materials, and their knowledge of student thinking and learning.	Under the supervision of the consulting teacher, the teachers of each Workshop make weekly sessions of study of the didactic units produced by a central team. This team has selected nuclear learning from the study plan and has written four units for each course. Each unit is a proposal of approximately five classes, mathematically and didactically grounded, so that the teacher can lead a learning process in the classroom.
One member of the team teaches the research lesson while fellow teachers and other observers collect data on student learning and thinking.	All the teachers who participate in the Workshop put into practice the proposal contained in the didactic units four times during a school year. Some of these classes are observed by the consulting teacher or by the technical chief of each School (academic director). They can also be videotaped.
The team discusses the lesson during a discussion session.	The consulting teacher organizes feedback workshops (devolution), both at school and communal levels, in which the classes are discussed and analysed.
The lesson is refined for the next teaching. Then the “teach - discuss - refine” cycle repeats.	The authors of the unit collect information through the follow-in process in order to reformulate the didactic units in their next versions.
At year-end the lesson planning team compiles a report on the findings and outcomes of their research.	Teachers who participate in the workshop are evaluated through tests to determine the progress of their mathematical and didactic knowledge during the year. The consulting teachers are also evaluated by means of tests but, in addition, they have to write a proposal report for teacher training.

Both LS and LCW are oriented to develop teacher knowledge across activities that lead to the improvement of teaching and learning in the classroom, to a better understanding of student thinking and to generate in teachers the need towork in a collaborative way. In LS this process is named "professional learning", whereas LCW refers to it as "professional development" or as "teacher's training".

In both models it has been difficult to explain to the administration of the educational system the principal purpose of the work that is proposed to teachers.

With regard to LS, we can mention Wang-Iverson and Yoshida (2005):

> The term lesson study, translated from the Japanese jugyokenku, has led to the myth that it means studying and improving a lesson until it is perfect (page 152).
>
> It is not easy to garner support for a long term effort designed to produce deep but incremental improvement from a district office under the pressure to rapidly raise test scores (page 40).

In relation to LCW, a document signed by an authority of the Ministry of Education, "Unsolved Problems and Proposals in Primary Education" (Sotomayor, 2006) states:

> It is necessary to produce didactic units for the whole year, once we have the model LEM. In the course of two years the whole school year must be covered, both in language and in mathematics, from Kinder to Fourth Grade (page 2).

The promoters of both strategies, in contraposition to the mentioned statements, consider as an instance of professional learning the work that teachers make in the cycle, comprising:

- planning (with the support of the didactic units, in the case of LCW)
- implementing and observing
- discussing and reflecting (devolution, for LCW)

In relation to LS, we mention again Wang-Iverson and Yoshida (2005):

> Lesson study is the core process of professional learning that Japanese teachers use to continually improve the quality of the educational experiences they provide to their students… It played a key role in transforming teaching from the traditional "teaching as telling" to "student cantered approach to learning" (page 3).
>
> Lesson study is a form of long-term teacher-led professional learning… and then use what they learn about student thinking and hatsumon (asking a question to stimulate student curiosity and thinking) to become more effective instructors (page 152).

With regard to LCW, in several documents in which the strategy is described, we find:

> On studying the didactic units, to implement them and carry out its later analysis, the teachers experiment and think about their own practice, extend and deepen their own mathematical knowledge living even successive fails, they value their children's possibilities of learning and they progress in the appropriation of a

methodology to plan, to manage and to evaluate productive processes of mathematical learning. (Espinoza, 2006)

Teachers use the didactic and mathematic tools acquired in the communal workshop to analyze the process (of teaching in the classroom) and the learning of the children (Espinoza, 2006).

A last dimension in which we are interested comparing LS and LCW is related to the participation of external agents on the teacher's team.

In LS the team invites an external expert to "collaborate with them to enhance content knowledge, guide the thinking about student learning and support the team's work" (Wang-Iverson and Yoshida, 2005, page 4). In this case, the expert provides his own theoretical frame.

In LCW we are working based on a specific theoretical approach (Chevallard, 1999). This approach considers the mathematical activity as the study of articulated problem fields. The lessons proposed in the didactic units are planned based on some outcome learning that have been selected from the national curriculum.

It is necessary to identify the mathematical tasks involved in these learning, which are presented to the students in the shape of problems. The techniques they will use spontaneously to explore the problematic situation are anticipated. Children will be allowed to make mistakes and stimulated to look for ways of overcoming them, on their own responsibility.

Along the sequence of classes the mathematical task, or its conditions of accomplishment, are modified in order to let the pupils experiment the need to find new techniques. By means of collective discussions they identify, among the techniques that emerge, the most effective ones. These techniques are practiced repeatedly, to generalize their appropriation in the classroom.

The problem that arises is the one of justifying the functioning of the recently adopted techniques, and then it becomes necessary to make explicit and to give a name to the underlying mathematical knowledge.

The sequence of lessons culminates with a systematization of the new knowledge, which is related to the previously acquired learning.

A Didactic Unit for the Learning of Fractions

The didactic unit that was used to plan the lesson that we will analyze later in the paper was designed for the Fourth Year of the Primary School. It is called: "Comparing the results of equitable and exhaustive distributions of fragmentable objects" (Espinoza and others, 2005).

The core learning of this unit is to acquire the idea that fractions are numbers that make possible the quantification of quantities in situations in which the natural numbers turn out to be insufficient.

The purposes of this didactic unit are to: (1) establish the need of the fractions as numbers, (2) relate the study of fractions to that of division in the field of natural numbers, and (3) extend the exploration, in order to compare fractions that result from distributions of objects of the same form and size.

The chosen context is the equitable and exhaustive distribution of a set of fragmentable objects (chocolate bars) among a group of people (children). The problem is to quantify the part that is distributed equally to each child. In this case, the fractions emerge when the number of objects to distribute is not a multiple of the participants' number. A second problem is to compare the quantities given to each participant in two different distributions. In this case, the object of the study is the order property in the field of the fractional numbers.

The didactic strategy consists of generating four lessons, with each lesson 90 minutes in length, in which a mathematical task is proposed to the students under different conditions, with the aim that the sequence of situations promotes the evolution of their knowledge.

The fundamental mathematical task is to quantify the result of an equitable and exhaustive distribution of fragmentable objects. The objects are square or rectangular, and they can be represented by pieces of paper.

The conditions of the distribution are [are the following conditions presented to students as unknowns, or is the following description for the benefit only of the reader?]:

- In the first class **1** object is distributed among **p** people, having p equal to 2, 4 or 8.
- In the second class **n** objects are distributed among **p** people, having **n < p** and p equal to the quantities of the first class, adding 3 and 6.
- In the third class **n** objects are distributed among **p** people, having **n > p** and p equal to the quantities of the second class.
- In the fourth class the relation between **n** and **p** can be anyone.

In connection to the techniques, in the first class they divide the paper that represents the object by mean of folds and cuts and write how much each person receives, using the fractional notation. Since they only can obtain unitary fractions, a second mathematical task is proposed to compare unitary fractions that correspond to the same object (a whole) distributed among different quantities of persons. Using techniques of visual inspection or overlapping the pieces of paper, they conclude that when the number of persons increases, the size of the part that each one receives diminishes. They deduce a criterion for the comparison of unitary fractions.

In the second class they also use the techniques of dividing by mean of folds and cuts but they already begin to anticipate the result of a distribution by means of reasoning: to distribute 3 objects among 4 persons each object is divided into 4 equal parts and you give 1/4 to each person. Since there are 3 objects, each person will receive 1/4 + 1/4 + 1/4, or ¾ [if students are just beginning to learn about fractions, how do they know how to add them already?]. This time, the task of comparing results of distributions appears as a comparison of fractions with the same numerator. For instance, the distribution of 2 chocolates among 4 persons and among 6 persons leads to the comparison of 2/4 with 2/6, which comes down to comparing 1/4 with 1/3, applying the criterion formulated in the first class.

In the third class, since n > p, we can expect the following:

- they distribute complete objects first, or that they make the division n:p and, when they obtain the rest (r) lower than p, they use the techniques of the first or of the second class, according to r = 1, or > 1. The result of the distribution will be a natural number (the whole quocient of n:p) plus a fraction less than 1 (r/p)
- they use the same techniques of the second class: to anticipate that it is possible to divide every object into as many parts as people. In this case the result of the distribution will be a fraction higher than 1, called also "improper" (n/p).

In the fourth class they will put into practice the same techniques used in the previous classes, since the tasks and their conditions are the same.

Analysis of an Observation of the Third Class.

The class[2] was conducted in May, 2005 by a teacher who was taking a course named "Curricular Appropriation" on Fractions, Decimals and Proportionality, at the University of Santiago de Chile. This course was taught by the team of authors of the didactic units LEM. As an assignment, this teacher had to design a didactic unit based on the structure of the LEM units. Since she was working with fourth grade students, she asked for authorization to put into practice the unit of fractions that we have described. Before

[2] This class, observed from its record in video, is described in the Appendix.

beginning, she had several interviews with one of her teachers in order to better understand the logic of this unit.

In the initial moment of this class, the teacher illustrates the mathematical tasks that the pupils carried out in the previous two classes: share of a rectangular object among p people and of n objects among p people, with $n < p$. She uses folding techniques without exposing them. She emphasizes the results and the fractional notation: 1/4 and 3/4.

In the central moment of the lesson, the teacher proposes a case where n is a multiple of p. In this case, the problem is solved by division with no remainder, and the result is greater than 1.

The mathematical work of the pupils then follows. This is announced by writing the problem in the blackboard and labelling it as a "challenge". It is a question of a case where $n > p$ and n is not a multiple of p.

The children work in teams of four. They have squares of paper, which they can manipulate in order to express their reasoning. Both the children and the teacher use only the folds, not the cuts, as they work with the papers that represent the objects that need to be distributed among the students. This can be due to the fact that the folds turn out to be sufficient to understand the mechanics and the result of the distributions, but we also can assume a matter of economy in the use of the material, so it can be reused.

During the sharing of ideas, the teacher contrasts the results of two techniques used by the pupils where both of them are correct:

- To distribute first the whole numbers according to the model of division of natural numbers and to divide the objects corresponding to the rest, so that the distribution is complete. The result is registered as a whole number plus a fractional number less than 1. To distribute the rest, if this one is 1, they use the technique used in the first class, and if it is different from 1, they use the technique corresponding to the second class.
- To divide each object in p equal parts and to assign to each person as many parts as there are objects, that is, n parts. The result is registered as n/p.

The teacher focuses the group discussion on the question whether the results are or aren't equivalent, without addressing the techniques used by the pupils. In the case of erroneous techniques (to divide every object in n equal parts), she listens to its description but she does not comment on them.

Referring to the objects that are supposedly going to be distributed, both the teacher and the children use the attribute of "whole numbers", for they are complete, not yet fragmented. The same term is used during other moments to designate the result of a distribution as "2 wholes plus 1/4". In the latter case, the word "whole number" alludes to a property of number 2, which distinguish it from the second term of the sum, which would be a "fraction". A slide takes place between both meanings, which may facilitate the comprehension of the "whole" term as an attribute of a number, due to the analogy between "2 whole numbers" and "2 whole bars of chocolate", but later on it will be necessary distinguish between the two statements.

As they each receive a worksheet, the children continue working in teams. The first task consists of a distribution of n among p, where n is a multiple of p. The division between natural numbers, as a resource to carry out this task, should have been learned before the study of this unit. Nevertheless, some children who try to divide with pencil and paper don't manage to reproduce the learned skill. On the other hand, the technique of distribution of n objects among p delimited spaces used by other children, though slow and rudimentary (they distributed one by one), turns out to be successful.

The second task on the worksheet consists of a distribution of n among p, and where $n < p$. Before determining the result of the distribution, as in the previous task, the teacher asks the children to guess if the result will be more or less than 1.

During the sharing of ideas after completing worksheet, the teacher considers the intervention of a pupil who says that in the first task it is necessary to do a division. We warn again that she emphasizes the result of the division, without specifying the techniques used to obtain it.

In responding to the second task, a pupil replies that they divided the n objects in halves, they distributed 1/2 to every p and what remained was divided in halves (1/4) and also distributed. The teacher listens attentively to this statement, but she does not comment on it.

In general terms, it should be noted that during this class the teacher generates working spaces in which she allows different techniques to emerge in the hands of the pupils, but at the moment of summarizing the achievement, she focuses the discussion on the obtained results, instead of on an analysis of the used techniques.

During the closing moment, carried out during additional time corresponding to the playtime, the conclusions boil down to if the result of a distribution is more or less that 1, as n is more or less than p, leaving out other different, possible conclusions of the work made in this class.

Testimony of the Teacher that Conducted the Class.

In an interview held four months later, this teacher referred to her learning in the course of "Curricular Appropriation" and, especially, to her experience of having put into practice the didactic unit on fractions:

> In the LEM units the planning comes very well constructed. Nonetheless, one has to work. It is not just a matter of copy. One has to study the unit to know what step is going to be given, what work is going to be done, and to adapt it to the reality of one's course. The unit of fractions helped me to raise another type of problems to my pupils. And they could solve them. The unit served me as a guide, because one can have an immense castle but if one does not work well, it could crumble down.
>
> I learned to have a clear notion of the task, the mathematical task that is going to be made by the child. When the task remains diffuse, the child loses time because she or he does not know what he or she is going to do. If the teacher clearly understands the task, the child does not lose time.
>
> I learned to give the children more work space during the class. I am enchanted by the way at which I work now, because the children are eager to participate. It is not important if they are wrong. If they are wrong I leave them, during a suitable time. Or they continue to work on the problem at home .
>
> I have now a passionate interest about the things that children say. With the unit, I could work by other ways and means, and watch what happens with the pupils. The children get enthusiastic, they think. They can draw conclusions, and they feel comfortable when they do it. They go back and advance, in agreement to what they have concluded previously. They are discovering things. They value the opinion of their classmates.
>
> I wouldn't be able to return and give the classes the way I did before. They were so boring, so square. I was imposing the learning. Everything was given, was made. In fractions you had to show them the little cake, the little apple. This is a 1/2, I wrote, without opening up possibilities for them to think, to go further.
>
> The implementation of a didactic unit means more work. But eventually it is less work, because the children learn more. They realize by themselves that 1/2 is equal to 2/4. They like to work with the fractions, relate them to other topics. I feel that they have learned.

Conclusions

The comparative analysis between Lesson Study and LEM Communal Workshops leads to the conclusion that both are powerful strategies to improve the educational practice and, at the same time, to generate processes of professional learning for teachers, which guarantee a higher stability of the changes achieved in their performance, with regard to other strategies.

One of the principal differences between Lesson Study and LEM Communal Workshops is that Lesson Study assumes a higher degree of autonomy of the teachers' team who work together, with regard to external experts. Thus, in the model of Lesson Study it corresponds to the teachers to choose the topic that they will work on and to plan a class. In LEM Communal Workshops, the teachers receive a quite well structured proposal of planning, which corresponds to a sequence of several classes. On the basis of this proposal, the teachers organize brief processes of study that culminate with a test to evaluate what the pupils have learned.

In this paper we provide evidence that indicates that teachers who use the LEM didactic units, after having studied them together with other colleagues, are able to manage their classes in a different way from the habitual one, opening spaces in order that their pupils carry out mathematical work during the class and take part in the construction of knowledge that correspond to their study plan.

However, in the extent which the teachers appropriate the mathematical tools and didactics contained in the LEM strategy, they are acquiring a higher grade of autonomy in their daily planning work. Paradoxically, the study, application and later commentary of very specific proposals, contained in the didactic units, lead the teachers to advance in a process of appropriation of what is necessary for them not "not to impose the knowledge on the pupils" and "to give them space in order that they work in the classroom, make mistakes, think and draw conclusions, " as reported by the teacher whose class we have analyzed.

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APPENDIX

1. Information about the VTR

Title: Sharing equally fragmentable objects

Topic: Comparison of fractions as a result of sharing equally fragmentable objects

Producer: LEM USACH Project, 2005. Headmaster: Dra. Lorena Espinoza. Faculty of Sciences. USACH, Chile.

Context: Curricular Appropriation course on Fractions, Decimals and Proportionality. Imparted by: Dr. Joaquim Barbé, Prof. Francisco Cerda and Prof. Fanny Waisman. 2005.

Video recorder: Prof. Francisco Cerda

Video editors: Alfredo Carrasco and Francisco Cerda

Teacher: Isabel Becerra

School: Colegio Altair. Comuna Padre Hurtado. Santiago.

Grade: Fourth Year of Primary School

Date: May, 2005

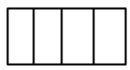
2. Description of the Observed Class.

The teacher begins, in the *initial moment*, with an inventory of the activities carried out in the previous two classes.

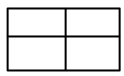
She presents 1 cardboard rectangle, she says "it is a whole" and folds it in 4 equal parts to simulate 1 chocolate that is distributed among 4 people (task of the first class). Every part is designated as 1/4.

Then, she presents 3 rectangles and folds each of them in 4 equal parts to simulate a distribution of 3 chocolates among 4 people (task of the second class). A student answers the question about how they would make it: "I would divide each chocolate into 4 parts, and I would give 3 pieces to each person". The teacher makes the folds and writes 1/4 in each part, that is to say, 4 times in each rectangle. A child writes in the blackboard the result of the distribution: 3/4.

It draws our attention the fact that she makes 3 parallel folds in the first rectangle:



On the other hand, in the other 3 rectangles she makes two perpendicular folds:



Though the rectangles are of the same form and size, nobody questions the fact that the same quantity of chocolate (1/4) is represented surfaces that are not congruent.

In the *central moment* the teacher proposes a distribution of 12 chocolates among 3 friends. She writes 12:3 = 4, and she comments that each child receives 4 “full”[3] chocolates.

Then she writes a "challenge" in the blackboard:

9 chocolate bars are distributed among 4 friends

¿How much chocolate does each one receive?

Children are assigned to teams of four. The teacher distributes 9 squares of paper to each group, and she allows them to work freely.

We observe students using different techniques to accomplish the proposed task. The recording allows us to distinguish among the work of three groups.

Group 1. We can see a child in great concentration, with his two hands in front, moving his fingers as if he was counting them. Then he explains to his classmates: "2 for each one and the bar that remains is divided in 4 pieces" He makes two perpendicular folds in a square to obtain 4/4. He says: "each one receives 2 wholes and 1/4". Then he explains: “for you, 2, for me, 2 ... there are 8 bars. It remains 1: 1/4, 1/4...” He makes the gesture of distributing, folding the paper but without cutting it.

Group 2. A girl distributes 2 squares for each person inher group. She folds the ninth square obtaining 4 equal parts, and she simulates distributing 1 part to each one (she doesn’t cut it).

Group 3. A girl proposes to divide each chocolate into 4 parts and to give one of these parts to each person. Thus, each person would receive 9/4 of the chocolate bar.

In this group another girl argues that each person will receive 2 bars and 1/4 of 1 bar, following the same reasoning observed in the previous groups.

In another group they fold each square to obtain 9 equal parts.

The teacher listens to the children who divided each square into 9 equal parts, but doesn’t comment on their technique.

The teacher organizes a summary where she confronts two techniques:

- To distribute first the whole objects and then to divide the remaining object. The result is registered on the blackboard as: 2 + 1/4.
- To divide each object into 4 equal parts and then to distribute all 36 resultant parts. The result is registered as: 9/4.

The teacher asks if it is the same thing: 2 + 1/4 and 9/4.

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[3] In spanish, she says: “enterito”, using the same word that we use for whole number (número entero).

To show the second procedure, the teacher takes 9 squares, each one folded into 4 equal parts, and she indicates one of these parts as she counts them, to verify that they are 9/4.

Some children argue that it is the same thing, because with 4/4 they make 1 whole (a bar of chocolate), with 8/4 they make 2 wholes and with the last 1/4 they complete 2 wholes and 1/4. They never work with cut parts to show this equivalence.

Later they work on individual worksheets. The teacher allows them to continue the team work.

The first activity proposes a distribution of 42 bars of chocolate among 6 children. They have to anticipate if each child will receive more or less than a bar of chocolate and have to write with numbers the amount of chocolate each child will receive.

A few children try to make the division 42:6, but they do not remember the procedure. They say "2 in 6 fits 3 times" and they write 3. Then they say "4 in 6 fits once" and they write 1. So, they write 31. Since it seems to be too much, they invert it, leaving 13.

In another group they decide to do the distribution with objects. They put their pencils together until they have 42. They share them in 6 groups. A child says: “this way we are going to finish tomorrow!”, but the girl who is sharing continues doing it. Finally they count the pencils of each group and say: “7!”.

The children then work on another distribution of 5 objects among 6 people, with the same questions.

The teacher organizes a summary, asking for the result of the first distribution. They give the answer: 7. Some children say that they have divided and others that 6 times 7 is 42. They answer that each child gets more than 1 chocolate.

As for the distribution of 5 among 6, the pupils say that each person gets less than 1 bar. A pupil explains that in his group they divided all 5 chocolates in halves, with which they would obtain 10/2. They gave a half to each of 6 persons, and then they divided all 4 remaining halves to distribute again among the 6 persons... The teacher listens but doesn’t comment on the technique that they used.

During the closing, already out of the time of the class, the teacher asks them to draw conclusions:

"How much corresponds to each person if the quantity of objects to be distributed is bigger than the amount of people? More than 1 or less than 1?" The children answer: "More than 1"

"And if the amount of objects is smaller than that of people? ", the teacher asks. The children answer that it would be less than 1.

3. A workshop for teachers.

1. Watch the video and comment on it freely.
2. Questioning.

   This phase deals with teachers solving problems related to the topic approached in the class and analyzing the techniques that they used and the mathematical and didactic knowledge that they have employed. If it is necessary, they complement their knowledge.

   Problem 1. In a meeting 17 people decide to order pizzas so that each person can eat 1/6 of a pizza. How many pizzas do they have to order?

   Problem 2. In another meeting 24 people order 5 pizzas of the same type of those of the previous meeting. They distribute them in equitably and completely. Determine if in this case every person will eat more or less pizza than in the previous meeting.

   Problem 3. Establish a sequence and explain it in order to present it to a fourth grade class, presenting the following tasks:

   To distribute 5 chocolates among 3 children

   To distribute 1 chocolate among 6 children

   To distribute 14 chocolates among 7 children

   To distribute 2 chocolates among 4 children

3. To watch the video again and to stop it to discuss:

   Initial moment:

   To identify the mathematical tasks.

   To justify the equivalence between 1/4 obtained by 3 parallel folds and by two perpendicular folds in a rectangle of paper.

   Central moment:

   To identify the mathematical tasks.

   To identify the techniques used by the children to solve the problem of distribution of 9 among 4.

   To justify the equivalence between 2 + 1/4 and 9/4, and to comment on the way in which it was managed by the teacher in the observed class.

   To identify the techniques used by the children to solve the problem of distribution of 42 among 7.

To propose a reaction, on the part of the teacher, to the technique described by a pupil to distribute 5 among 6 (to divide by the half).

Closing moment:

To determine what other aspects might be included in the closing of this class.

4. To compare the comments made during the first and the second time they have seen the video.
5. To draw conclusions based upon the proposal contained in the video and upon the way in which they habitually teach this topic.
6. **Homework:** To write a paragraph on the relation that the pupils can establish between division in natural numbers and fractions, as quantification of parts of a whole object.