Mathematical problem solving / Mathematical modeling

(Typhoon)

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Mathematical problem solving and mathematical modeling – Typhoon

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Introduction

The formation of typhoon

http://www.cwb.gov.tw/V7e/knowledge/encyclopedia/me024.htm

Introduction

The air over the tropical seas is warmer due to the heat of sunlight; therefore more sea water is evaporated that makes the air over the tropical seas warm and wet and expands due to its higher temperature, which will in turn reduces its density. This combined with the low wind force in the equatorial region provide a perfect recipe for air upward motion.

Introduction

When the air rises, cooler air from the surroundings will flow in to fill in the space; later this refilling air will be heated and lifted, gradually forming a circulation of air. This process, called convection, will cause the whole column of air to become lighter and lower in density, creating a low pressure system called tropical depression.

Introduction

Since the sun shines directly to north of the Equator during summer, the southeast trade wind from the Southern Hemisphere will be transformed into a southwest monsoon when they pass across to the Northern Hemisphere. When a southwest monsoon meets with a northeast trade wind in the Northern Hemisphere, the two will converge and bring the air upward, enhancing the convection effect. Due to the difference of the wind directions and the nature of the southwest monsoon and the northeast trade wind, they will disturb each other when they meet and create a vortex.

Tropical cyclones of different intensity are given different names.

Tropical cyclones are classified in accordance with the World Meteorological Organization's. With effect from 2009, a new classification is used and consists of 6 categories as follows:

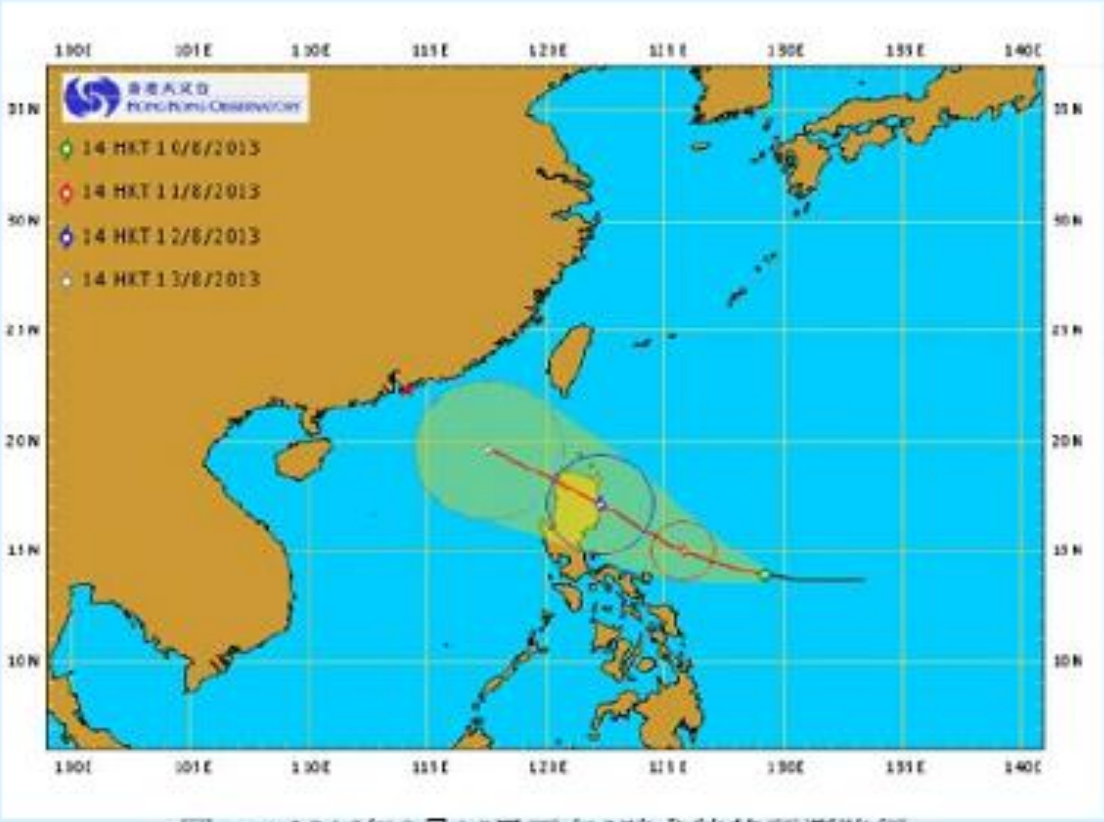
Tropical Cyclone Classification	**Maximum sustained winds near the centre (km/h)**
Tropical Depression (TD)	<63
Tropical Storm (TS)	63-87
Severe Tropical Storm (STS)	88-117
Typhoon (T)	118-149
Severe Typhoon (ST)*	150-184
Super Typhoon (SuperT)*	185 or above

*New categories starting 2009

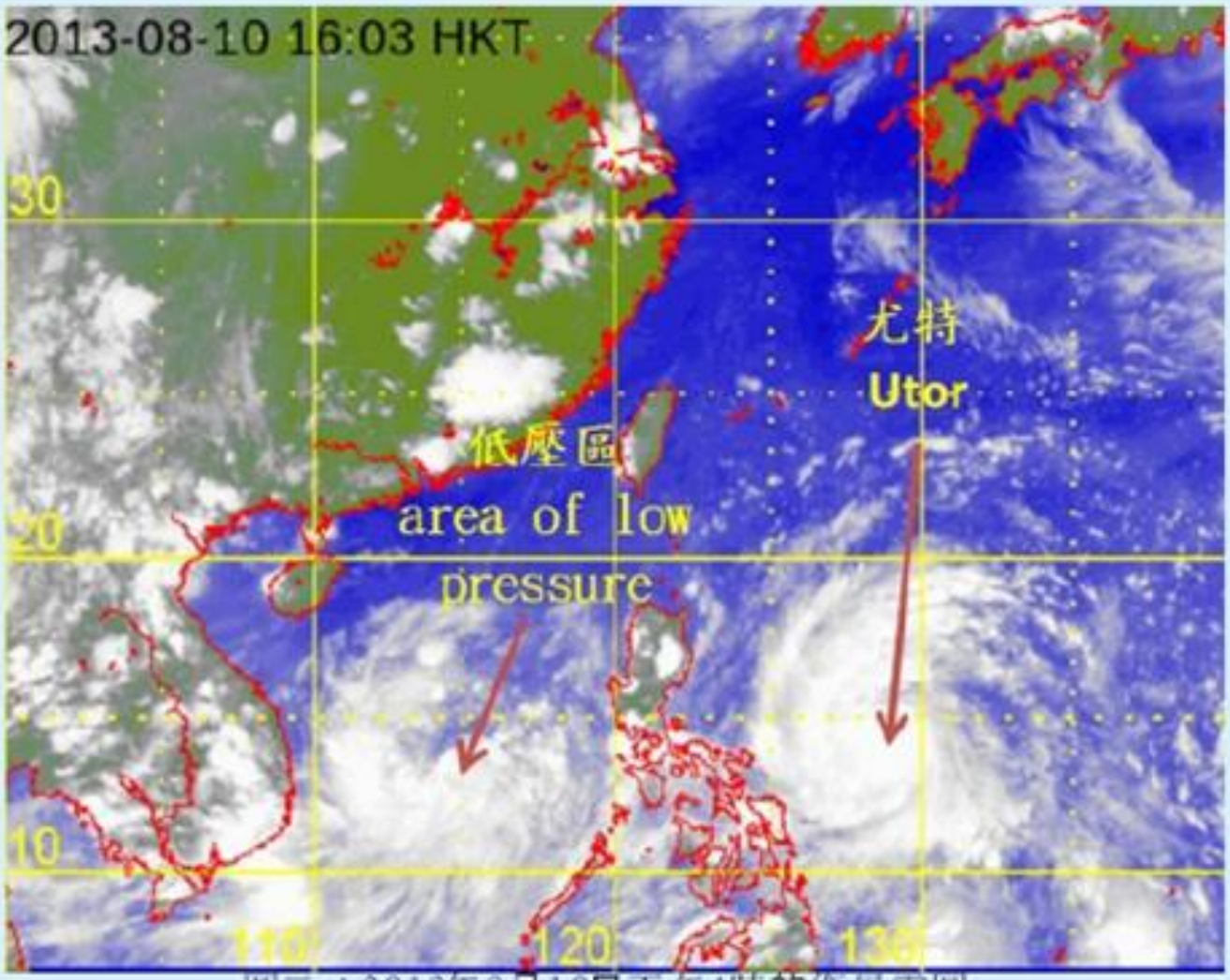
http://www.weather.gov.hk/education/edu01met/01met_tropical_cyclones/ele_typhoon5_e.htm#faq

熱帶氣旋尤特未來動向仍有變數
〔2013 年 08 月 10 日〕

熱帶氣旋尤特今天 (2013年8月10日) 在菲律賓以東的太平洋上迅速增強，並採取西北偏西的方向移動。按照預測路徑 (圖一)，尤特會大致移向呂宋北部，並會在下週初進入南海。在今日下午4時，尤特離香港超過1,800公里。普遍來說，尤特預測位置的偏差會隨預測時間增加而增大。此外，一個低壓區正在南海徘徊 (圖二)。若尤特進入南海時，亦會受到這低壓區的影響以致動向可能出現更多變化。現時說尤特下星期如何影響香港是言之尚早。市民應密切留意天文台對尤特的最新預測。



圖一：2013年8月10日下午2時尤特的預測路徑



圖二：2013年8月10日下午4時的衛星雲圖

Background information

Circumstances of the earth = 40,000km

Circumstances of the earth = 360×60= 21600 knouts

Exercise

(i) From the picture taken by satellite, estimate the location of the centre of the typhoon Utor

Center of Utor at = _____

Answer: 14N, 129E

(ii) From the picture taken by satellite, estimate the radius of the typhoon Utor

Radius of Utor = _____

Answer : (diameter from 10N to 18N, radius = 4×60 =240 knouts = 444 km)

(iii) From the picture taken by satellite, estimate location of Hong Kong

Location of HK = _____

Answer: 22.5N, 114E

(iv) Estimate the distance of Utor from Hong Kong (assume that the circumstances of the Longitude is always 21600 knouts)

Distance Location of HK = _____

Answer: 22.5N - 14N = 8.5, and 129E-114E = 15

Correspondence distance = 8.5×60 = 510 knouts and 15×60 = 900knouts

Based on the Pythgroian theroem

Estimated distance = $\sqrt{510^2 + 900^2} = 1034$ knouts

(v) Estimate the speed of Utor (from the graph, Utor is at 15N, 126E on 11/8/2103, and at 20N, 117.5E on 11/8 2013)
The distance travelled in 48 hours is about (20-15)N, (126-117.5)E

Moved 5N and 8.5E, 9.86×60 knouts = 592 knouts =1066 km.

Speed = 1066/48 = 22km per hour

Given the location of the typhoon Utor,
fill in the boxes
predict the movement of the typhoon.

Tropical Depression UTOR

Time	Location		Type
14:00 HKT p 09 August 2013	13.7 N	133.5 E	Tropical Depression
20:00 HKT 09 August 2013	13.7 N	132.5 E	Tropical Depression
02:00 HKT 10 August 2013			Tropical Storm
08:00 HKT 10 August 2013	13.7 N	130.4 E	Tropical Storm
11:00 HKT 10 August 2013	13.8 N	129.9 E	Severe Tropical Storm
14:00 HKT 10 August 2013			Typhoon

Tropical Depression UTOR

Time	Location		Type
14:00 HKT p 09 August 2013	13.7 N	133.5 E	Tropical Depression
20:00 HKT 09 August 2013	13.7 N	132.5 E	Tropical Depression
02:00 HKT 10 August 2013	13.7 N	131.6 E	Tropical Storm
08:00 HKT 10 August 2013	13.7 N	130.4 E	Tropical Storm
11:00 HKT 10 August 2013	13.8 N	129.9 E	Severe Tropical Storm
14:00 HKT 10 August 2013	13.9 N	129.2 E	Typhoon

20:00 HKT 10 August 2013	14.0 N	128.2 E	Typhoon
02:00 HKT 11 August 2013	14.2 N	127.1 E	Typhoon
08:00 HKT 11 August 2013			Typhoon
11:00 HKT 11 August 2013	14.8 N	125.4 E	Severe Typhoon
14:00 HKT 11 August 2013	15.1 N	124.8 E	Severe Typhoon
17:00 HKT 11 August 2013			Severe Typhoon
20:00 HKT 11 August 2013	15.5 N	123.5 E	Super Typhoon
23:00 HKT 11 August 2013	15.8 N	122.8 E	Super Typhoon
02:00 HKT 12 August 2013			Super Typhoon

20:00 HKT 10 August 2013	14.0 N	128.2 E	Typhoon
02:00 HKT 11 August 2013	14.2 N	127.1 E	Typhoon
08:00 HKT 11 August 2013	14.5 N	125.9 E	Typhoon
11:00 HKT 11 August 2013	14.8 N	125.4 E	Severe Typhoon
14:00 HKT 11 August 2013	15.1 N	124.8 E	Severe Typhoon
17:00 HKT 11 August 2013	15.3 N	124.1 E	Severe Typhoon
20:00 HKT 11 August 2013	15.5 N	123.5 E	Super Typhoon
23:00 HKT 11 August 2013	15.8 N	122.8 E	Super Typhoon
02:00 HKT 12 August 2013	16.2 N	122.2 E	Super Typhoon

05:00 HKT 12 August 2013	16.4 N	121.5 E	Severe Typhoon
08:00 HKT 12 August 2013	16.6 N	120.8 E	Severe Typhoon
11:00 HKT 12 August 2013			Severe Typhoon
14:00 HKT 12 August 2013	17.4 N	118.8 E	Severe Typhoon
17:00 HKT 12 August 2013	17.6 N	118.5 E	Severe Typhoon
20:00 HKT 12 August 2013			Severe Typhoon
23:00 HKT 12 August 2013	18.0 N	117.4 E	Severe Typhoon
02:00 HKT 13 August 2013	18.2 N	116.8 E	Severe Typhoon
05:00 HKT 13 August 2013			Severe Typhoon

05:00 HKT 12 August 2013	16.4 N	121.5 E	Severe Typhoon
08:00 HKT 12 August 2013	16.6 N	120.8 E	Severe Typhoon
11:00 HKT 12 August 2013	17.0 N	120.0 E	Severe Typhoon
14:00 HKT 12 August 2013	17.4 N	118.8 E	Severe Typhoon
17:00 HKT 12 August 2013	17.6 N	118.5 E	Severe Typhoon
20:00 HKT 12 August 2013	17.7 N	118.1 E	Severe Typhoon
23:00 HKT 12 August 2013	18.0 N	117.4 E	Severe Typhoon
02:00 HKT 13 August 2013	18.2 N	116.8 E	Severe Typhoon
05:00 HKT 13 August 2013	18.3 N	116.0 E	Severe Typhoon

08:00 HKT 13 August 2013	18.3 N	115.3 E	Severe Typhoon
05:00 HKT 13 August 2013	18.3 N	116.0 E	Severe Typhoon
08:00 HKT 13 August 2013			Severe Typhoon
11:00 HKT 13 August 2013	18.5 N	114.8 E	Severe Typhoon
14:00 HKT 13 August 2013	18.9 N	114.1 E	Severe Typhoon
17:00 HKT 13 August 2013			Severe Typhoon
20:00 HKT 13 August 2013	19.3 N	113.6 E	Severe Typhoon
23:00 HKT 13 August 2013	19.5 N	113.3 E	Severe Typhoon
02:00 HKT 14 August 2013			Severe Typhoon

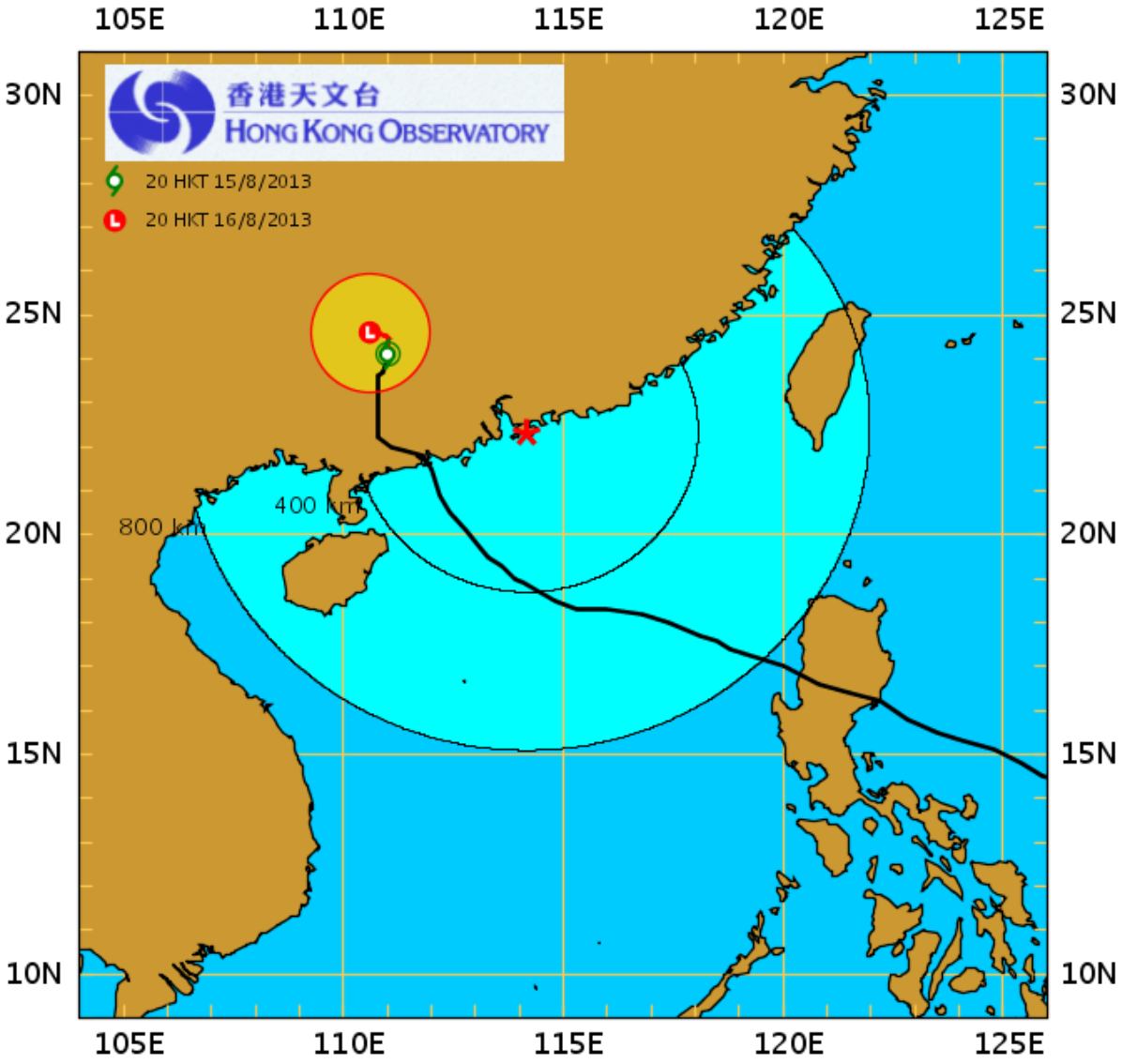
08:00 HKT 13 August 2013	18.3 N	115.3 E	Severe Typhoon
05:00 HKT 13 August 2013	18.3 N	116.0 E	Severe Typhoon
08:00 HKT 13 August 2013	18.3 N	115.3 E	Severe Typhoon
11:00 HKT 13 August 2013	18.5 N	114.8 E	Severe Typhoon
14:00 HKT 13 August 2013	18.9 N	114.1 E	Severe Typhoon
17:00 HKT 13 August 2013	19.0 N	113.9 E	Severe Typhoon
20:00 HKT 13 August 2013	19.3 N	113.6 E	Severe Typhoon
23:00 HKT 13 August 2013	19.5 N	113.3 E	Severe Typhoon
02:00 HKT 14 August 2013	19.7 N	113.1 E	Severe Typhoon

05:00 HKT 14 August 2013	20.1 N	112.8 E	Severe Typhoon
08:00 HKT 14 August 2013	20.5 N	112.4 E	Severe Typhoon
11:00 HKT 14 August 2013			Severe Typhoon
14:00 HKT 14 August 2013	21.5 N	112.0 E	Typhoon
17:00 HKT 14 August 2013	21.8 N	111.8 E	Typhoon
20:00 HKT 14 August 2013			Typhoon
23:00 HKT 14 August 2013	22.0 N	111.1 E	Severe Tropical Storm
02:00 HKT 15 August 2013	22.2 N	110.8 E	Severe Tropical Storm
05:00 HKT 15 August 2013			Severe Tropical Storm

05:00 HKT 14 August 2013	20.1 N	112.8 E	Severe Typhoon
08:00 HKT 14 August 2013	20.5 N	112.4 E	Severe Typhoon
11:00 HKT 14 August 2013	20.9 N	112.2 E	Severe Typhoon
14:00 HKT 14 August 2013	21.5 N	112.0 E	Typhoon
17:00 HKT 14 August 2013	21.8 N	111.8 E	Typhoon
20:00 HKT 14 August 2013	21.9 N	111.4 E	Typhoon
23:00 HKT 14 August 2013	22.0 N	111.1 E	Severe Tropical Storm
02:00 HKT 15 August 2013	22.2 N	110.8 E	Severe Tropical Storm
05:00 HKT 15 August 2013	22.6 N	110.8 E	Severe Tropical Storm

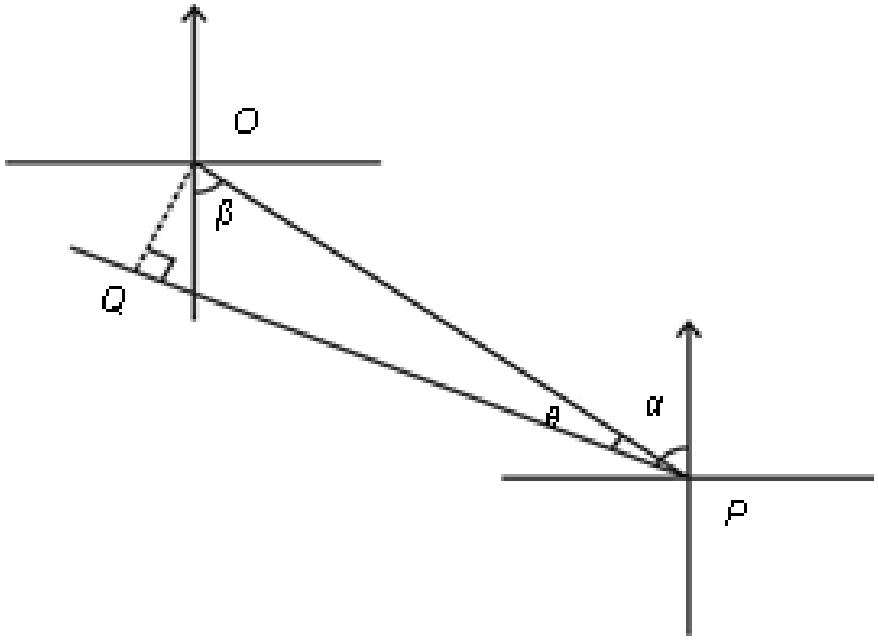
08:00 HKT 15 August 2013	23.1 N	110.8 E	Tropical Storm
11:00 HKT 15 August 2013	23.3 N	110.8 E	Tropical Storm
14:00 HKT 15 August 2013			Tropical Depression
17:00 HKT 15 August 2013	23.7 N	110.9 E	Tropical Depression
20:00 HKT 15 August 2013	24.1 N	111.0 E	Tropical Depression

08:00 HKT 15 August 2013	23.1 N	110.8 E	Tropical Storm
11:00 HKT 15 August 2013	23.3 N	110.8 E	Tropical Storm
14:00 HKT 15 August 2013	23.6 N	110.8 E	Tropical Depression
17:00 HKT 15 August 2013	23.7 N	110.9 E	Tropical Depression
20:00 HKT 15 August 2013	24.1 N	111.0 E	Tropical Depression


105E
110E
115E
120E
125E
30N
25N
20N
15N
10N
香港天文台
HONG KONG OBSERVATORY
20 HKT 15/8/2013
20 HKT 16/8/2013
400 km
800 km

★ 香港 Hong Kong

Mathematical problem solving or mathematical modeling



P = location of the Typhoon

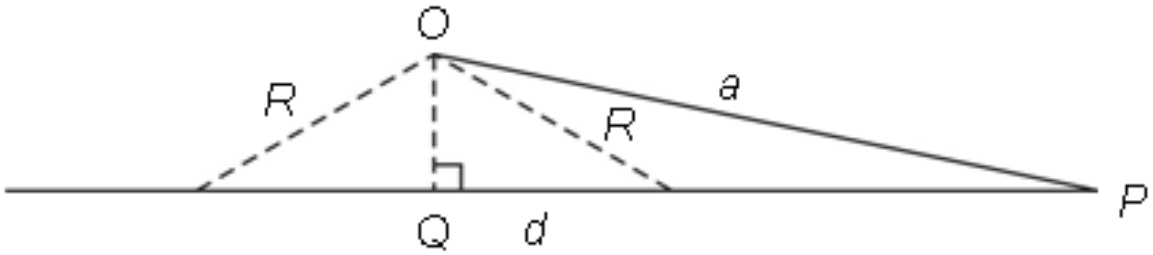
Q = closest encounter

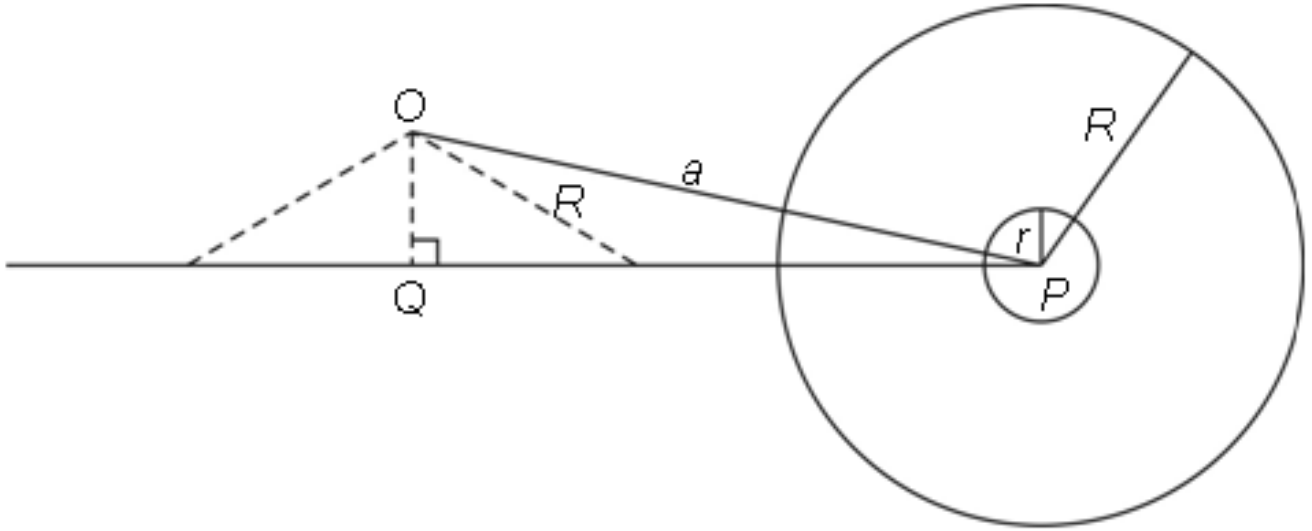
O = the city

a = distance of the typhoon P from city O.

v = speed of movement of the Typhoon

Arrival time of the centre of the Typhoon to city = $t = \frac{a\cos\theta}{v}$


O
R
a
R
P
Q
d


O
R
a
R
r
P
Q

Let r = radius of the centre of typhoon

R = the radius of influence of the typhoon

Then the time that started the influence of the typhoon is done by

$d =$ _____

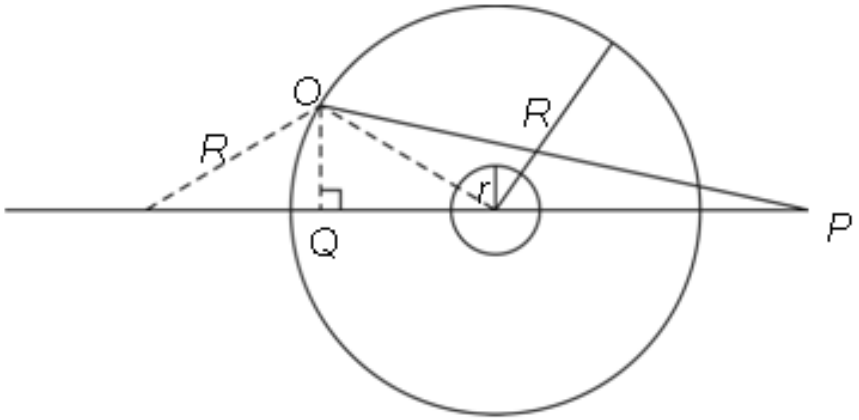
$T =$ _____

The time that the typhoon influence the area

= _____

The time that the typhoon leave the area

= _____



Consider the center of the typhoon:
By the time

$t_1 =$ _____, the center enters the region of O.

And by the time

$t_2 =$ _____, the center leaves the region of O.

The time interval that affect the city O = $t_2 - t_1 = \dfrac{2\sqrt{r^2 - a^2 \sin^2 \theta}}{v}$

And $t =$ _____ is the time when the center is closest to city O.

Answer

Let r = radius of the centre

R = the radius of influence of the typhoon

Then the time that started the influence of the typhoon is done by

$$d = \sqrt{R^2 - a^2 \sin^2 \theta},$$

$$T = \frac{d}{v} = \frac{\sqrt{R^2 - a^2 \sin^2 \theta}}{v}$$

The time that the typhoon influence the area

$$= \frac{a \cos\theta - \sqrt{R^2 - a^2 \sin^2 \theta}}{v}$$

The time that the typhoon leave the area

$$= \frac{a \cos\theta + \sqrt{R^2 - a^2 \sin^2 \theta}}{v}$$

Consider the center of the typhoon:

By the time

$t_1 = \dfrac{a\cos\theta - \sqrt{r^2 - a^2\sin^2\theta}}{v}$, the center enters the region of O.

And by the time

$t_2 = \dfrac{a\cos\theta + \sqrt{r^2 - a^2\sin^2\theta}}{v}$, the center leaves the region of O

The time interval that affect the city O = $t_2 - t_1 = \dfrac{2\sqrt{r^2 - a^2\sin^2\theta}}{v}$

And $t = \dfrac{a\cos\theta}{v}$ is the time when the center is closest to city O.

Thank you