APEC-Chiang Mai International Conference IV:
Innovation of Mathematics Teaching and Learning through Lesson Study-
Connection between Assessment and Subject Matter –

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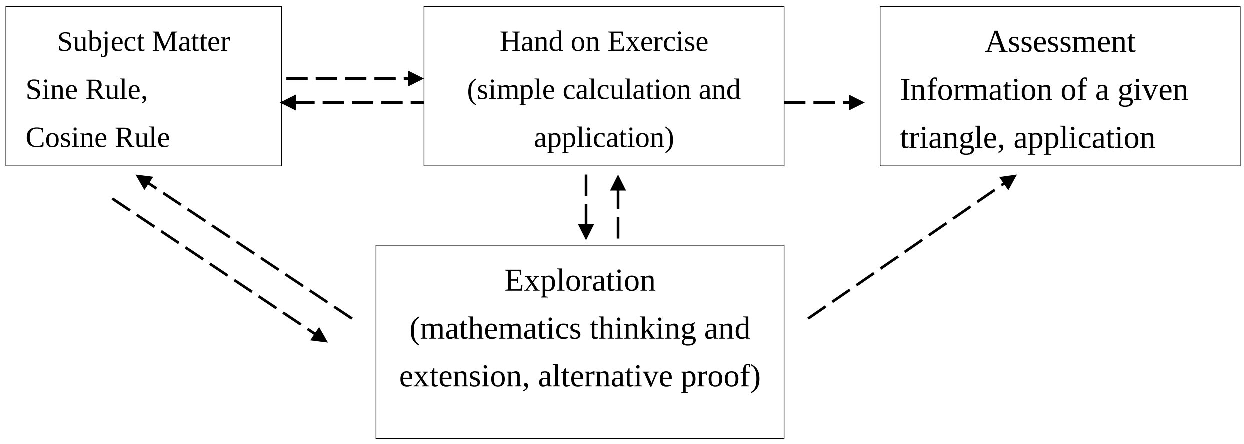
Report on using Sine Rule and Cosine Rule to demonstrate the connection between Assessment and Subject Matter

Introduction, Hand on assessment

It is difficult to understand mathematical concept in one single lesson. Hence, it is a good approach to work on the exercise on the topic and then through working the example to consolidate the concepts. Such working on the exercise allow us to pick up different pieces of concepts and consolidate them through the problems. It is the hand on working and assessment that help to build the concepts.

The paper would like to discuss the components of Sine Rule and Cosine Rule, its subject matter and the assessment items related, so that assessment items are used as a tools to enhance the advancement of the mathematical concepts.



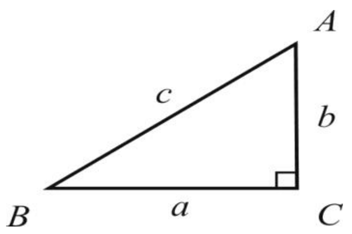
Subject matter in Sine Rule

Sine Rule Special case (right angle triangle)

Using a right angle, students can explore and obtain the following relations.

$\sin B = \frac{b}{c}$, $\sin A = \frac{a}{c}$ 。

$\Rightarrow \quad c = \frac{a}{\sin A} = \frac{b}{\sin B}$ 。



As sinC = 1, so it is trae that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

The next step is to establish the formula "$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$"」 for all triangles.

In the following proof, a related assessment question is attched.

Sine Rule Proof 1

Using the relation of areas of triangles, we have

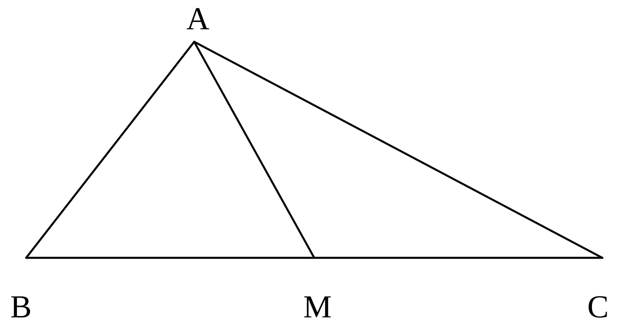
$\frac{1}{2}ab\text{Sin}C = \frac{1}{2}bc\text{Sin}A = \frac{1}{2}ac\text{Sin}B$

$\Rightarrow \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$,

This is the simplest way to establish the formula.

Sine Rule Proof 1b (based on the logic of Proof 1)

Using the median AM of △ABC.



Area $\triangle$ABM = $\frac{1}{2}$(AB)(BM)sinB

Area $\triangle$ACM = $\frac{1}{2}$(AC)(CM)sinC。

As both areas are equal and BM = CM, c sinB = b sinC。

$\Rightarrow \frac{c}{\sin C} = \frac{b}{\sin B}$。

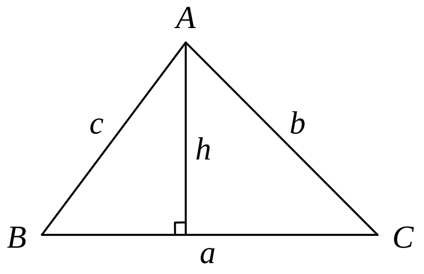
Sine Rule Proof 2

Using the height h of the triangle as a reference,

sinB = $\frac{h}{c}$, sinC = $\frac{h}{b}$,

$\Rightarrow$ c sinB = bsinC

$\Rightarrow \frac{c}{\sin C} = \frac{b}{\sin B}$



Sine Rule Proof 2b (based on the logic of the Proof 2)

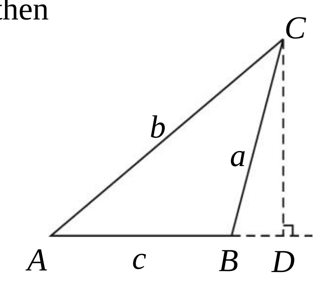
ΔABC with obstue angle B.

draw $CD \perp AB$, meeting the extension of AB at D, then

$\frac{CD}{b} = \sin A, \Rightarrow CD = b\sin A$;

$\frac{CD}{a} = \sin(180^\circ - B) = \sin B, \Rightarrow CD = a\sin B$,

Hence $b\sin A = a\sin B$, and $\frac{a}{\sin A} = \frac{b}{\sin B}$



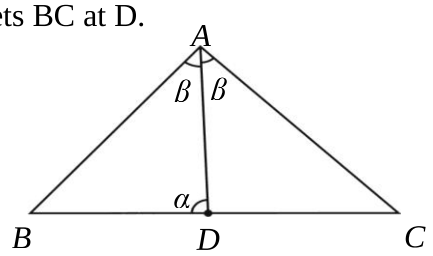
Similarly, $\frac{b}{\sin B} = \frac{c}{\sin C}$,

Hence $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Question：

In ΔABC, angle bisector AD of $\angle A$ meets BC at D.

Prove that $\frac{BD}{DC} = \frac{AB}{AC}$



As in diagram, using Sine Rule in ΔABD and ΔCAD,

$\frac{BD}{\sin\beta} = \frac{AB}{\sin\alpha}$ ①

$\frac{DC}{\sin\beta} = \frac{AC}{\sin(180^\circ - \alpha)} = \frac{AC}{\sin\alpha}$ ②

①÷②, $\Rightarrow \dfrac{BD}{DC} = \dfrac{AB}{AC}$

Connection and Exploration

$\dfrac{a}{\sin A}$ **=** $\dfrac{\text{b}}{\text{sinB}}$ **=** $\dfrac{\text{c}}{\sin C}$ **= ?**

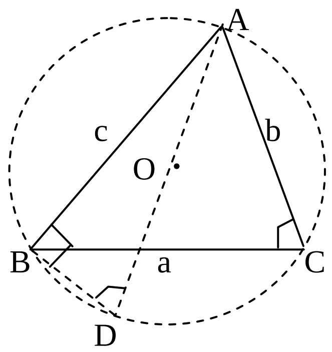
What is the value of the $\dfrac{a}{\sin A} = \dfrac{\text{b}}{\text{sinB}} = \dfrac{\text{c}}{\sin C}$?

Statement：   In ΔABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

$\dfrac{a}{\sin A} = \dfrac{\text{b}}{\text{sinB}} = \dfrac{\text{c}}{\sin C}$ = 2R, but how this is done?

Let O be the centre of a circle inscribing ΔABC,
Connect AO and extend AO to meet the circle at D
AD = diameter.

$\Rightarrow$ $\angle DBA = 90^\circ$, $\angle BDA = \angle ACB$



In right angle triangle ABD,

$AB = AD \cdot \sin\angle BDA = AD \cdot \sin C = 2R\sin C$

$\Rightarrow$ $c = 2R\sin C$

Similarly, $a = 2R\sin A$ ， $b = 2R\sin B$

$\Rightarrow$ $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

Discussion and assessment

Only two of the three relations in sine rule is independent

1. From $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ ， $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$, we have $\dfrac{c}{\sin C} = \dfrac{a}{\sin A}$

2. From $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$, there may exist positive number c and angle C, so that

   $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ or such result does not hold.

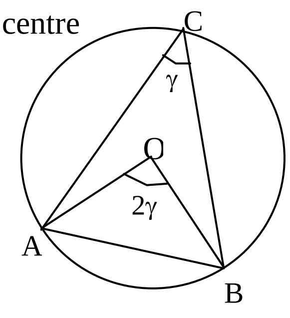
Connection and exploration

The deduction of mathematical concepts from given problem

The following result is given to student and they are asked to related them to Sine Rule.

From "Element, Theorem 20, book 3"

In a circle, the angle subtends by the same arc at the centre is twice the angle subtend at the circumference.

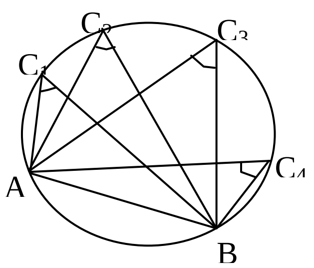


Extension 1

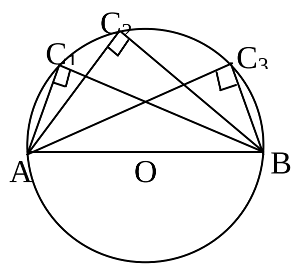
The angle subtended at the circumference are the same (theorem 21 of Element)；

Extension 2

Angle subtended at the centre is a right angle.

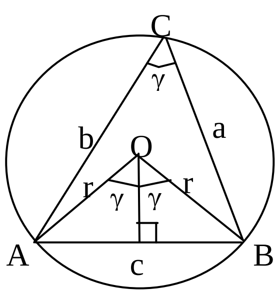


(1) Theroem 21

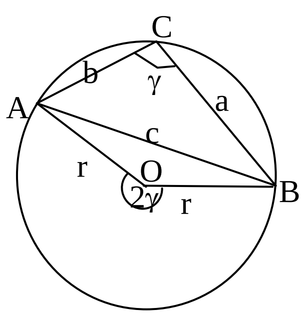


(2) Angle subtended are right angle

Using the results, the class can prove the Sine Rule.



Sine Rule (Acute angle)



Sine Rule (Obtuse angle)

When γ is acute, centre of the circle is lies inside the triangle. ΔABC inscribed in the circle with radius r,centre O. And $\angle AOB = 2\angle ACB = 2\gamma$. From O draws AB, the perpendicular bisector, then $\sin\gamma = (\frac{c}{2})/r$ ， $\Rightarrow \frac{c}{\sin\gamma} = 2r$ $\Rightarrow \frac{a}{\sin\alpha} = \frac{\beta}{\sin\beta} = \frac{c}{\sin\gamma} = 2r$	When γ is obtuse, the centre of the circle is outside the triangle. The arc AB is more than half of the circumference, hence the angle at O of ΔAOB is $\gamma' = 360^\circ - 2\gamma$ 。 From O draws AB, the perpendicular bisector, then $\frac{\sin\gamma'}{2} = (\frac{c}{2})/r$ 。 But $\frac{\sin\gamma'}{2} = \sin(180^\circ - \gamma) = \sin\gamma$ ， Hence $\frac{c}{\sin\gamma} = 2r$.

Cosine Rule

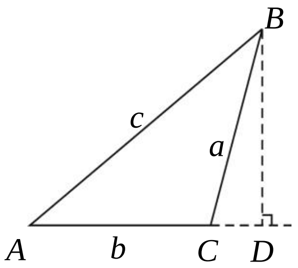
Proof of Cosine Rule 1

Extension of Pythagoras Theorem

Using Pythagoras Theorem

$(a \,\mathrm{Sin}C)^2 + (-a \,\mathrm{Cos}\, C + b)^2 = c^2$ 。

$\Rightarrow (a)^2 + (b)^2 - 2ab\mathrm{Cos}C = c^2$ 。



Result	
In ΔABC , we have	$c^2 = a^2 + b^2 - 2ab\cos C$ $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$

Cosine Rule Proof 2

Through A, construct AD ⊥ BC and meet at D

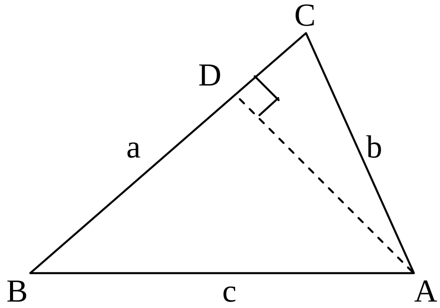
then $CD = b\cos C$ ，

$BD = a - b\cos C$ 。

Hence $c^2 - BD^2 = AD^2 = b^2 - CD^2$

$\Rightarrow c^2 - (a - b\cos C)^2 = b^2 - b^2\cos^2 C$

$\Rightarrow c^2 = a^2 + b^2 - 2ab\cos C$



Similarly, $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

Cosine Rule Proof 3

$b\cos A + a\cos B = c$ ， $\Rightarrow$ $cb\cos A + ca\cos B = c^2$

$c\cos B + b\cos C = a$ ， $\Rightarrow$ $ac\cos B + ab\cos C = a^2$

$a\cos C + c\cos A = b$ ， $\Rightarrow$ $ab\cos C + bc\cos A = b^2$

From the above formula,

$a^2 + b^2 = ac\cos B + bc\cos A + 2ab\cos C \ = \ c^2 + 2ab\cos C$

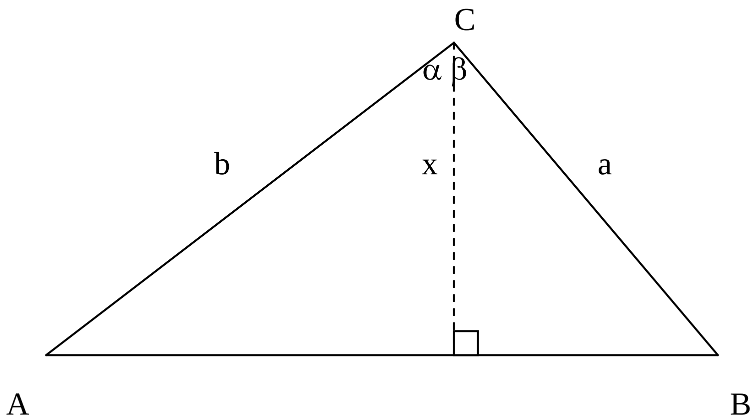
$\Rightarrow \ c^2 = a^2 + b^2 - 2ab\cos C$

Similarly,

$a^2 = b^2 + c^2 - 2bc\cos A$,

$b^2 = a^2 + c^2 - 2ac\cos B$.

Connection (Heron Formula)



From the diagram, obtain $x^2 = \dfrac{[(b+c)^2 - a^2][a^2 - (b-c)^2]}{4c^2}$ 。

Area of triangle = $\dfrac{1}{2}cx$

= $\dfrac{1}{2}cx \ \dfrac{\sqrt{[(b+c)^2 - a^2][a^2 - (b-c)^2]}}{2c}$

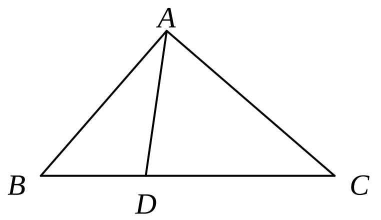
= $\dfrac{1}{4}\sqrt{[(b+c)^2 - a^2][a^2 - (b-c)^2]}$ 。

Which is the same as $\sqrt{\mathrm{p}(\mathrm{p}-a)(\mathrm{p}-b)(\mathrm{p}-c)}$, where p = $\dfrac{1}{2}(a+b+c)$ 。

From the following format, a number of assessments can be generated.

Question Format：

In ΔABC , D is a point on BC

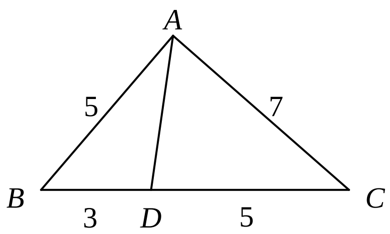


Question：

In ΔABC , a point D on $\overline{BC}$ satisfy

$\overline{AB} = 5$, $\overline{AC} = 7$, $\overline{BD} = 3$, $\overline{CD} = 5$,

then $\overline{AD}$ =?



In ΔABC ， $\cos B = \dfrac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8} = \dfrac{1}{2}$

And in ΔABD , $\overline{AD}^2 = \overline{AB}^2 + \overline{BD}^2 - 2\overline{AB} \cdot \overline{BD} \cos B$

$$= 25 + 9 - 15 = 19$$

$$\Rightarrow \overline{AD} = \sqrt{19}$$

Question：

In ΔABC , point D on $\overline{BC}$ satisfy $\overline{AB} = 6$, $\overline{AC} = 3\sqrt{2}$, $\angle BAD = 30^\circ$ and $\angle CAD = 45^\circ$, find $\overline{AD}$.

Let $\overline{AD} = x$, Area $\Delta ABD + \Delta ACD = \Delta ABC$

$$\Rightarrow \frac{6x}{2}\sin 30^\circ + \frac{3\sqrt{2}x}{2}\sin 45^\circ = \frac{6 \times 3\sqrt{2}}{2}\sin 75^\circ$$

$$\Rightarrow \frac{3x}{2} + \frac{3x}{2} = \frac{9\sqrt{2}}{4}\left(\sqrt{6} + \sqrt{2}\right)$$

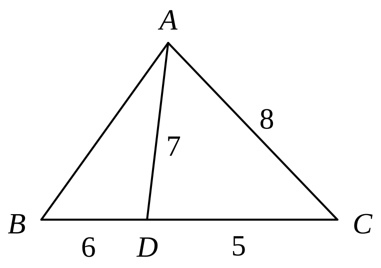
$$\Rightarrow x = \left(\sqrt{3} + 1\right)$$

Question：

In ΔABC, $\overline{AC}=8$, $\overline{BC}=11$,

D is a point on C and $\overline{AD}=7$, $\overline{BD}=6$.

Find $\overline{AB}$



$\overline{CD}=5$, solving C from ΔACD, and obtain $\overline{\text{AB}}$ from ΔABC.

In △ACD, $\cos C=\dfrac{8^2+5^2-7^2}{2\times 8\times 5}=\dfrac{1}{2}$

In △ABC, $\overline{AB}^2=8^2+11^2-2\times 8\times 11\cos C=64+121-88=97$

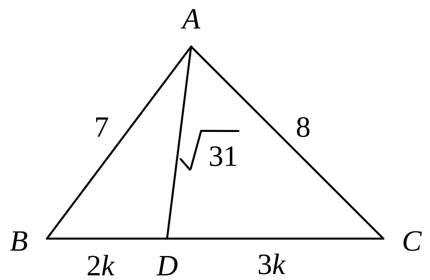
$\Rightarrow \overline{AB}=\sqrt{97}$

Question：

In △ABC, $\overline{AB}=7$, $\overline{AC}=8$,

D is a point on BC, and $\overline{AD}=\sqrt{31}$, $\overline{BD}:\overline{CD}=2:3$

Find $\overline{BC}$.



Let $\overline{BD}=2k$, $\overline{CD}=3k$.

By common angle B, the triangle ΔABD and ΔABC both has one angle and three sides, Using cosine rule, B and k are solved.

In ΔABD, $\cos B=\dfrac{(2k)^2+7^2-\left(\sqrt{31}\right)^2}{2\times 2k\times 7}=\dfrac{2k^2+9}{14k}$

In ΔABC, $\cos B=\dfrac{(5k)^2+7^2-8^2}{2\times 5k\times 8}=\dfrac{5k^2-3}{14k}$

$\Rightarrow \dfrac{2k^2+9}{14k}=\dfrac{5k^2-3}{14k}$

$\Rightarrow k=2$

$\Rightarrow \overline{BC} = 5k = 10$ 。

Assessment, there are three types of questions.

A Two sides and two angles, find the third side.

Q	Length		Angles		Answer
1	$\overline{BD} = 50$	$\overline{BC} = 200$	$\angle ADB = 60^\circ$	$\angle ACB = 30^\circ$	$\overline{AB} = 50\sqrt{7}$
2	$\overline{AB} = 6\sqrt{2}$	$\overline{AC} = 2\sqrt{3}$	$\angle BAD = 30^\circ$	$\angle CAD = 45^\circ$	$\overline{AD} = 3\sqrt{2}$
3	$\overline{BD} = 4$	$\overline{CD} = 4\sqrt{3}$	$\angle ABC = 45^\circ$	$\angle ACB = 30^\circ$	$\overline{AD} = 4$
4	$\overline{AC} = 7$	$\overline{BD} : \overline{CD} = 2:3$	$\angle BAD = 45^\circ$	$\angle CAD = 60^\circ$	$\overline{AB} = 2\sqrt{6}$

B Three sides and an angle given, find another length.

Q	Length			Angles	Answer
1	$\overline{AB} = 8$	$\overline{AC} = 6\sqrt{3}$	$\overline{AD} = 6$	$\angle CAD = 30^\circ$	$\overline{BD} = 3 + \sqrt{37}$
2	$\overline{AB} = 4$	$\overline{AC} = 7$	$\overline{BD} = 4$	$\angle ABC = 60^\circ$	$\overline{CD} = -2 + \sqrt{37}$
3	$\overline{AD} = 5$	$\overline{AC} = 8$	$\overline{BD} = 7$	$\angle CAD = 60^\circ$	$\overline{AB} = 2\sqrt{21}$
4	$\overline{AB} = 4$	$\overline{AC} = 2\sqrt{10}$	$\overline{CD} = 4$	$\angle ABC = 45^\circ$	$\overline{BD} = 6\sqrt{2} - 4$
5	$\overline{AB} = 3$	$\overline{AC} = 5$	$\overline{BD} = \overline{CD}$	$\angle BAC = 120^\circ$	$\tan \angle BAD = 5\sqrt{3}$
6	$\overline{BD} = 3$	$\overline{DC} = 6$	$\overline{AB} = \overline{AD}$	$\angle BAD = \angle CAD$	$\cos \angle BAD = \frac{3}{4}$

C Four lengths are given, find the fifth length

Q	Length				Answers
1	$\overline{AB} = 7$	$\overline{AD} = 13$	$\overline{BD} = 7$	$\overline{CD} = 8$	$\overline{AD} = 7$
2	$\overline{AB} = 7$	$\overline{AD} = 3$	$\overline{BD} = 5$	$\overline{CD} = 2$	$\overline{AC} = \sqrt{7}$
3	$\overline{AB} = 5$	$\overline{AC} = 5$	$\overline{BD} = 2$	$\overline{AD} = 4$	$\overline{CD} = \frac{9}{2}$

Assessment

Which Rule to use, Sine Rule or Cosine Rule?

The important part of mathematics is thinking, usually students learn a theorem and

then they apply the theorem at some given situations.
For example, the following two exercises require students to use Cosine Rule and Sine Rule.

Question：
In ΔABC, $a=5$, $b=8$, $c=7$. Find $\angle C =$.

From the conditions given, student need to relate that three sides given satisfies the requirement of the cosine rule.

And $\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{5^2+8^2-7^2}{2\times5\times8} = \dfrac{1}{2}$

$\Rightarrow \angle C = 60^\circ$

Question：
In ΔABC, $\angle A = 105^\circ$, $\angle B = 45^\circ$, $b = 8\sqrt{2}$. $c =$?

From the conditions given, two angles can only related to the using of Sine Rule,

$C = 180^\circ - A - B = 30^\circ$

Hence $\dfrac{c}{\sin C} = \dfrac{b}{\sin B} \Rightarrow \dfrac{c}{\sin 30^\circ} = \dfrac{9\sqrt{2}}{\sin 45^\circ}$

$\Rightarrow \dfrac{c}{\frac{1}{2}} = \dfrac{8\sqrt{2}}{\frac{\sqrt{2}}{2}}$

$\Rightarrow c = 8$

In the following, students need to think of how to use the two theorems.

Discussion

By $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$ ，we have $\dfrac{a}{b} = \dfrac{\sin A}{\sin B}$ 。

If B = 2C, then $\dfrac{b}{c} = \dfrac{\sin 2C}{\sin C}$ = $\dfrac{2\sin C\cos C}{\sin C}$ = 2cosC 。

$\Rightarrow$ the ratio of sides b and c is 2cosC.

Question：

In ΔABC, $b=8$, $c=5$, $\angle B=2\angle C$。

Find the value of $\cos\angle A$.

By $\angle B=2\angle C$, we have $A=180^\circ-3C$,

$\cos A=\cos(180^\circ-3C)=-\cos 3C$, the question is to find cosC.

The rest is to use Sine Rule.

$$\frac{b}{\sin B}=\frac{c}{\sin C}\Rightarrow\frac{8}{\sin 2C}=\frac{5}{\sin C}$$

$$\Rightarrow 5\sin 2C=8\sin C$$

$$\Rightarrow 10\sin C\cos C=8\sin C$$

$$\Rightarrow \cos C=\frac{4}{5}\quad(\text{as}\sin C\neq 0)$$

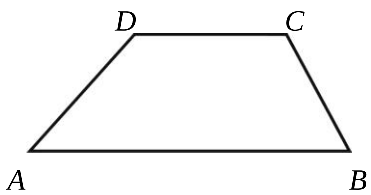
Hence $\cos A=\cos(180^\circ-3C)=-\cos 3C$

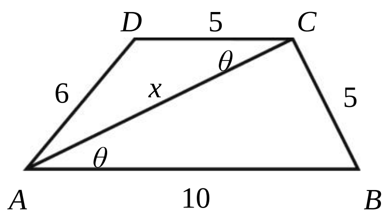
$$=-(4\cos^3 C-3\cos C)$$

$$=-\left[4\left(\frac{4}{5}\right)^3-3\left(\frac{4}{5}\right)\right]=\frac{44}{125}$$

Question：

In the trapzeum *ABCD*, $\overline{AB}//\overline{CD}$, $\overline{AB}=10$, $\overline{BC}=5$, $\overline{CD}=5$, $\overline{DA}=6$.
Find $\overline{AC}$.





From common side $\overline{AC} = x$

As $\overline{AB} // \overline{CD}$, let $\angle ACD = \angle CAB = \theta$ 。

Using Cosine Rule, $\cos\theta = \dfrac{x^2 + 5^2 - 6^2}{2 \times x \times 5}$ and $\cos\theta = \dfrac{x^2 + 10^2 - 5^2}{2 \times x \times 10}$

$\Rightarrow \dfrac{x^2 + 5^2 - 6^2}{2 \times x \times 5} = \dfrac{x^2 + 10^2 - 5^2}{2 \times x \times 10}$

$\Rightarrow 2(x^2 - 11) = x^2 + 75$

$\Rightarrow x^2 = 97$

$\Rightarrow x = \sqrt{97}$

Exploration

> Question：
>
> Prove that the sum of product of distances and the sine of the angle from a point inside a triangle to the three sides is a constant.
> That is, $h_a \sin A + h_b \sin B + h_c \sin C$ is a constant.

Let *P* any point inside ΔABC, and denote $AB = c$, $BC = a$, $CA = b$,
The distance from *P* to *a, b, c* are h_a、h_b、h_c .
Connect *PA, PB* and *PC* 。

$S_{\Delta PAB} + S_{\Delta PBC} + S_{\Delta PCA} = S_{\Delta ABC} = \Delta$

$\Rightarrow \dfrac{1}{2}ch_c + \dfrac{1}{2}ah_a + \dfrac{1}{2}bh_b = \Delta$

To allow the common part of *a, b, c* 中, using sine rule (2*R* is the diameter of the circle inscribe ΔABC)： $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

then $R\sin A \cdot h_a + R\sin B \cdot h_b + R\sin C \cdot h_c = \Delta$

$\Rightarrow h_a \sin A + h_b \sin B + h_c \sin C = \dfrac{\Delta}{R}$

Question：

The sides of ΔABC are a, b, c, and $\frac{b}{c-a}-\frac{a}{c+b}=1$, find the largest angle of ΔABC.

$$\frac{b}{c-a}-\frac{a}{c+b}=1$$

$$\Rightarrow \quad b(c+b)-a(c-a)=c^2+cb-ac-ab$$

From c > a, and $b^2+a^2+ab=c^2$, so c > b.

Uisng Cosine Rule, $\cos C=\frac{a^2+b^2-c^2}{2ab}=-\frac{1}{2}$,

$\Rightarrow$ C = 120°

Only Two of the three relations in the Cosine Rule are independent, from two of the three, the third relation could be deduced.

By $a^2=b^2+c^2-2bc\cos A$, and $b^2=c^2+a^2-2ca\cos B$

That is, $\cos A=\frac{b^2+c^2-a^2}{2bc}$, and $\cos B=\frac{c^2+a^2-b^2}{2ca}$

$$\because \ A+B+C=180^\circ$$

Then, $\cos C=\cos[180^\circ-(A+B)]=-\cos(A+B)$

$$=\sin A\sin B-\cos A\cos B$$

$$=\sqrt{1-\cos^2 A}\cdot\sqrt{1-\cos^2 B}-\cos A\cos B$$

$$=\sqrt{1-\left(\frac{b^2+c^2-a^2}{2bc}\right)^2}\cdot\sqrt{1-\left(\frac{c^2+a^2-b^2}{2ca}\right)^2}-\frac{b^2+c^2-a^2}{2bc}\cdot\frac{c^2+a^2-b^2}{2ca}$$

$$=\frac{2(a^2b^2+b^2c^2+c^2a^2)-(a^4+b^4+c^4)}{4abc^2}+\frac{a^4+b^4-c^4-2a^2b^2}{4abc^2}$$

$$=\frac{2(b^2c^2+c^2a^2)-2c^4}{4abc^2}$$

$$=\frac{a^2+b^2-c^2}{2ab}$$

$$\therefore \ c^2=a^2+b^2-2ab\cos C$$

By $a^2=b^2+c^2-2bc\cos A$, there may or may not exist angle B, such that $b^2=c^2+a^2-2ca\cos C$

Assessment (Sine Rule)

1	In ΔABC, $\sin^2 A + \sin^2 B = \sin^2 C$, prove that ΔABC is a right angle triangle.
2	In ΔABC, if acosA = bcosB, what kind of triangle is ΔABC?
3	In ΔABC, prove that $\frac{\sin A + \sin B}{\sin C} = \frac{a+b}{c}$ 。

Assessment, Cosine Rule

1	For ΔABC, prove that $a^2 + b^2 + c^2 = 2(bc\text{Cos}A + ac\text{Cos}B + ab\text{Cos}C)$ 。
2	Using Cosine Rule to prove The "sum of squares of the sides of a paralellgram" equals "the sum of the square of the diagaonls".

Mixed Assessment

1	In ΔABC, ∠A = 2∠B, prove that a = 2b cosB.
2	In ΔABC, ∠C = 2∠B, prove that $\frac{\sin 3B}{\sin B} = \frac{a}{b}$ 。
3	在ΔABC 中，sinA(cosB+cosC) = sinB + sinC, prove that ΔABC is a right angle triangle
4	In ΔABC, if $\frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C}$, prove that ΔABC is an equilateral triangle.
5	In ΔABC, $\frac{a^2 - (b-c)^2}{bc} = 1$, find ∠A.
6	In ΔABC, sinA = 2sinBcosC, show that ΔABC is issolsoles triangle.
7	The sum of two sides of a triangle is 10, the included angle is 60°, find the minimum perimeter of this triangle.

References

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