DEVELOPING MATHEMATICAL THINKING IN A PRIMARY MATHEMATICS CLASSROOM THROUGH LESSON STUDY: AN EXPLORATORY STUDY

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Abstract

This paper discusses an exploratory study that aims to develop mathematical thinking in a primary mathematics lesson. Although mathematical thinking is one of the significant components of Malaysian school mathematics curriculum, it was not explicitly implemented in many Malaysian schools due to time constraints and mathematics teachers' lack of understanding and awareness about mathematical thinking. In views of the importance of mathematical thinking, it was set up as one of the goals of a Lesson Study group existing in a Chinese primary school. Two lesson study cycles were carried out with a result of two mathematics lessons planned and observed. Five mathematics teachers participated in the study. Preliminary analysis shows that these mathematics teachers espoused that it was much easier to learn new teaching ideas such as developing mathematical thinking through Lesson Study collaboration. Initially, many teachers did not understand fully what mathematical thinking is and how to help pupils to develop this kind of thinking. After two lesson study cycles, these teachers have gained much more understanding and confidence in developing mathematics lessons that promote mathematical thinking. Nevertheless, time constraint and heavy workload remain their two main challenges to integrate any new teaching ideas and strategies.

Introduction

This paper discusses an exploratory study that attempts to develop mathematical thinking in a primary mathematics classroom through Lesson Study collaboration. A document analysis of the Malaysian primary and secondary mathematics curricula done earlier (Lim & Hwa, 2006) indicates that promoting mathematical thinking among Malaysian pupils is an intended goal but it was not explicitly spelled out in the syllabus. A literature search of local studies signify the need to have much more empirical study that focus on promoting mathematical thinking in the Malaysian classroom. Informal discussion with school mathematics teachers displayed that many mathematics teachers agreed to the importance of mathematical thinking and would like to promote mathematical thinking in their classrooms. But they are usually constrained by several issues and challenges such as (i) lack of clear understanding of mathematical thinking; (ii) lack of appropriate assessment tool that measure mathematical thinking and (iii) lack of know-how to promote mathematical thinking (Lim & Hwa, 2006).

In view of the importance of mathematical thinking and the potential of Lesson Study collaboration, an attempt was made to develop mathematical thinking as a goal of an existing Lesson Study group in a Chinese primary school. This Chinese Primary School situated in the centre of an urban area. It is a mini-size school consists of one

headmistress, one male teacher, ten female teachers, and 136 pupils. There are only 6 classes with one class for each grade. The Lesson Study group of this school consisted of 8 mathematics teachers were set up since January 2006. They have gone through three lesson study cycles in the year 2006, with the aim of enhancing mathematics teachers' content knowledge and their confidence in teaching mathematics in English language (detailed report see Goh Siew Ching, 2007).

In the following section, I will first explore the teachers' understanding of mathematical thinking; follow by a brief description of the exploratory study. This study comprises of three stages: (a) an introductory workshop on mathematical thinking; (b) first lesson study cycle; and (c) second lesson study cycle. To highlight how these teachers' attempt to develop pupils' mathematical thinking, parts of the two lesson plans collaboratively designed by the Lesson Study group will be used to elaborate together with video clips of the lessons observed.

Teachers' perceptions of mathematical thinking

To elicit mathematics teachers' understanding of mathematical thinking, a brief questionnaire was given to the 6 mathematics teachers and 5 non-mathematics teachers who attended the workshop. Analysis of their response show that majority of these teachers were not sure if they were ready to promote mathematical thinking in the classroom. The main reason was "teachers are not given enough resources to promote mathematical thinking in the classroom". All except two did not answer the question, "Are Malaysian teachers promoting mathematical thinking in the classroom?" The two who answered were also not sure "because they [mathematics teachers] merely convey the knowledge of doing or solving the problems of mathematics."

Out of the 11 teachers, two of them agreed that they understand what mathematical thinking is, two disagreed while others were not sure. Consequently, only two of them agreed that they know how to promote mathematical thinking in the classroom. To these teachers, mathematical thinking refers mainly to problem solving, involve creative and logical thinking, and require skills such as reasoning, analyzing and the use of mathematical symbols. One mathematics teacher believed that she has been incorporated mathematical thinking in her daily teaching although she did not explicitly mention it in class. For her, asking a lot of "why" questions and giving pupils a variety of questions to solve are ways of promoting mathematical thinking.

An exploratory study to promote mathematical thinking

In view of the importance of mathematical thinking and the lack of proper understanding of mathematical thinking among teachers, an exploratory study was proposed to promote mathematical thinking among mathematics teachers through Lesson Study collaboration.

An introductory workshop on mathematical thinking

On March 9, 2007, all the 11 teachers attended an introductory workshop on mathematical thinking. The main aim of the workshop was to expose these teachers to the concept of mathematical thinking and to propose some possible strategies to promote mathematical thinking in the classroom. These teachers were shown a videotaped Japanese classroom lesson of a Grade 4 mathematics topic on "prime and composite number". Before showing the video, the teachers were given the same classroom activity to experience. Ten cards of different designs were arranged in a specific way. Teachers were asked to observe the order of the designs and determine what the patterns or order represent. They were then asked to identify the rules and using these rules to arrange the successive two cards. The teachers seemed to enjoy this activity and some of them were able to come out with certain kind of rules.

Later, the teachers were shown the video lesson and asked to list out the characteristics of mathematical thinking that they observed in the lesson. The following list was the outcome:

- Activity based
- Pupil centred, active pupil participation
- Justifying, reasoning, argue, debating
- Extrapolating, extend to new situations
- Generalizing, evaluating
- Decision making
- Positive attitude – willing and eager to try
- Logical thinking, creative thinking etc

Based on the list, the teachers were encouraged to plan a mathematics lesson that promotes mathematical thinking through their Lesson Study group collaboration.

Teachers were encouraged to write their reflection after the workshop. Some teachers reported in their written journals that they have been practicing some of the above characteristics of mathematical thinking in their daily class teaching. However, many of them were not aware that these were elements of mathematical thinking. They espoused that they were keen to plan out a mathematics lesson that will help to develop mathematical thinking.

First Lesson Study cycle (22 March-27 April 2007)

Five mathematics teachers participated the first Lesson Study cycle. The topic chosen was "percentage". See Appendix I for a detail lesson plan. In this cycle, the teachers met four times: 3 meetings for discussion on lesson planning and one for teaching observation followed with reflection and discussion.

Second Lesson Study cycle (13 June-16 July 2007)

In the second Lesson Study cycle, the same five mathematics teachers participated. The topic chosen was "Time" for Grade 4 class. See Appendix II for a detail lesson plan. In this cycle, the teachers met five times: 4 meetings for discussion on lesson planning and one for teaching observation followed with reflection and discussion.

General outline of the lesson

Table 1 displays the general flow or outline of the two lessons. According to the participating teachers, this is also the common format of their normal mathematics lessons. However, small group activities are seldom carried out as it is time consuming. Instead, teachers tend to explain the related mathematical concepts with examples and then give a lot of questions for pupils to practice. Nevertheless, to develop mathematical thinking, they suggested the best way is to promote through small group activity. This is because small group activity will exhibit some characteristics of mathematical thinking, such as active pupil participation, encourage pupils to present and justify their answers, and promote logical and creative thinking.

Table 1: General flow or outline of the lesson

	Lesson 1	Lesson 2
Topic (grade level)	Percentage (Grade 5)	Time (Grade 4)
Learning outcome	Convert proper fraction to percentage	Addition and conversion of time in minutes and hours
Induction set	Represent information in fraction and percentage	Link to pupils' daily life experience: favourite TV programme
Step 1	Small group activity	Small group activity 1: jigsaw puzzle
Step 2	Pupil presentation	Pupil presentation
Step 3	Practice and discussion	Small group activity 2: jigsaw puzzle
Step 4	More practice and discussion	Pupil presentation
Closure	Enrichment exercise-worksheet as homework	Enrichment exercise-worksheet as homework

Developing mathematical thinking in Lesson 1

Pupils were divided into four groups. Each group was given 3 cards, labelled as M, S and E. To stimulate the interest of the pupils, the teacher has creatively linked the cards to M for Monkey; S (Snake) and E (Elephant). Pupils were asked to write down a number between 50-100 for card M; 20-50 for card S and less than 20 for card E.

Without any prior objective of what the number will indicate, pupils simply gave a number that suited the condition. Some wrote 40 over 50 (on card S); some wrote 60 over 100 (on card E). Initially it was planned that these numbers will represent the quiz scores for each group. For example, M stands for Mathematics quiz; S for science quiz and E for English quiz. The mathematics quiz has maximum score of 100; science quiz maximum score is 50 and English quiz maximum score is 20. The pupils were then asked to write their scores in fraction form and later convert to percentage. Finally teacher asked the pupils, "Which group has the best total score to be declared as the winner of the quiz competition? What is the best way to decide?"

This was planned in such a way, so that pupils will need to rationalize [using mathematical thinking] that they have to change the score from fraction form to percentage, so that the three scores can be compared to decide the winner.

However, as reflected by the teacher, Mr L, later in the discussion after teaching observation that he has forgotten this part of the lesson plan. He forgot to ask the pupils to decide which was the best total score. Instead he asked pupils to suggest the best score for each subject.

During the teaching observation, some pupils appeared to be rather unsure about the request of the teacher. One pupil came out to give 19 over 20. But very soon he realized his error and he changed his answer to 20/20. Similarly another pupil wrote 41/100 for mathematics quiz. It was then corrected by his friend to be 100/100. These pupils' answers show that some of them understood that the best score for each subject should be 100%. Nevertheless, it was a pity that the teacher failed to grip the opportunity to encourage more mathematical thinking among pupils, by asking pupils to justify their answers.

Developing mathematical thinking in Lesson 2

Lesson 2 aims to teach the Grade 4 pupils how to add and convert two quantities of time in minutes and hours. The teacher began the lesson by asking pupils' favourite television programme and the amount of time they used to watch these programme per week. This created a cheerful discussion as all pupils were keen to share what were their favourite television programmes. To make the calculation simple, the teacher limited the number of programme to only one per day. As there was no programme on Wednesday, a total of 6 programmes were watched per week. Since each programme was shown for 30 minutes, a total of 6 x 30 minutes which equal to 180 minutes or 3 hours was the total time of watching. This was a direct and simple calculation sum for the pupils. However, to promote mathematical thinking, the teacher challenged the pupils to suggest alternative methods. One girl proposed multiple additions. She then demonstrated her method in front of the class (see Figure 1). She added 30 minutes for six times and yielded the same answer of 180 minutes in total. This is an example of mathematical thinking because pupils were encouraged to show variation in methods of solving.

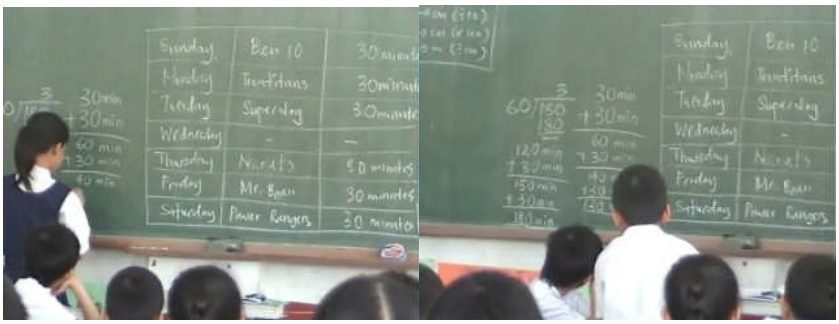

Figure 1: pupil show alternative method

In the second part of lesson 2, pupils were divided into 6 groups. Each group was given an envelope which contained two sets of question. Pupils were encouraged to discuss in group and to match every sheet of paper given to form a correct set of mathematical relationship. For example, match 3 sheets of paper as “45minutes + 50 minutes = 95 minutes”; or “35 minutes + 28 minutes = 1 hour 3 minutes”. All pupils were observed to participate actively and keenly in the given activity. Later, each group presented their solutions to the class. One pupil from each group was also asked to demonstrate their method of solving on the board.

To promote mathematical thinking, the teacher deliberately asked a lot of “why” questions to her pupils. For example, a girl pupil subtract directly 120 minutes from the total sum (see Figure 2), instead of the usual method of divide by 60 minutes. The teacher asked her to justify and the girl was able to explain that 120 minutes equals to 2 hours.

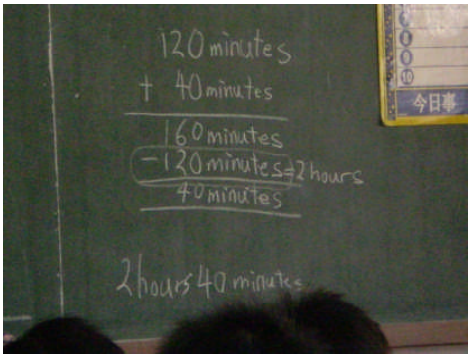

Figure 2

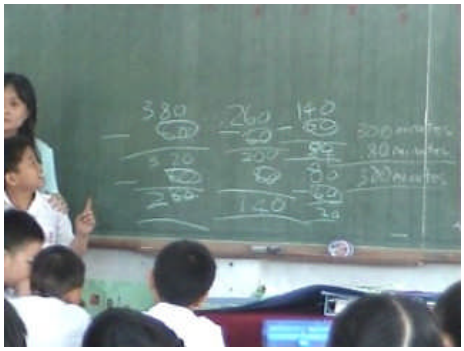

Figure 3

Similar to the first part of lesson 2, the teacher challenged frequently her pupils for alternative methods. For example, in relation to the equation: “300 minutes + 80 minutes = 6 hours 20 minutes”, both pupils displayed the same method of solving as “300 + 80 = 380” and then divide 380 minutes by 60 minutes to give the answer of 6 hours and 20 minutes. So, the teacher challenged her pupils, “Besides divide by 60, is there any other method of getting the answer?” One boy proposed, “minus!” The boy was then asked to demonstrate his method to the class. He displayed how to solve by multiple subtractions (Figure 3).

Taking this opportunity, the teacher also extended pupils’ mathematical thinking to new situations. The following dialogue demonstrates this point:

Teacher (T): Why do you circle all the 60 and 60?

Pupil (P): Because 60 minutes is one hour.

T: So you circle how many 60s here?

P: 6

T: it means how many hours?

P: 6 hours

……

The teacher also took this opportunity to ask the whole class.

T: so if take away four 60s means how many hours?

P (choral answer): 4 hours

T: how about eight 60s?

P: (choral answer) : 8 hours

Hence, it was observed that the teacher in lesson 2 was working very hard to incorporate mathematical thinking in her lesson. One also noticed that she was code-switching (using either English or Mandarin) to explain and to give instruction so as to ensure that all her pupils understand her teaching. The pupils of this class are made up of three races: Chinese, Indian and Malay. The majority of them do not understand English language very well.

Teachers' reflection

Immediately after each teaching lesson, all the five teachers and the researcher gathered to reflect and discuss. As part of the lesson study process, teachers were also encouraged to write out their reflections in their journals after every discussion and teaching observation. They were allowed to write using any language that they were comfortable with. Out of the five teachers, two of them wrote using English language, two wrote in Mandarin and one wrote in Malay language.

Teachers' Reflection on Lesson 1

The teacher who taught Lesson 1, Mr L expressed that he was rather nervous at the beginning because he was trying to recall and to follow what was planned in the lesson plan. He rated himself as 50% successfully achieved the objectives of the lesson. He was rather happy that even the 4 weakest students in his class seemed to pay attention today. He admitted that he changed what was planned in the lesson plan after the induction set.

The four teachers who observed lesson 1 expressed positive support and comments to Mr L. They contented that Mr L has clear and loud voice, very good rapport with his students, confident, patient and experienced. They also praised each other for preparing colourful power point presentation and worksheets.

One teacher, Ms S pointed that the instruction given by Mr L was rather confused. She saw many pupils did not know how to proceed, and she was rather worried at that time. Consequently, another teacher Ms M proposed that Mr L could have asked the pupils to solve based on one subject at a time and not all three subjects at the same time. Likewise, another teacher, Ms K reflected on herself that given that situation, she would quickly give examples and show to her pupils how to solve them. She was amazed that Mr L was very patient and waited patiently for his pupils to explore and to find out the answers by themselves.

When asked if they have incorporated mathematical thinking in that lesson, they all agreed that they have attained to a small extent. For example, when the teacher asked,

"what will be the percentage if there are only 10 questions?" This kind of question encouraged pupils to extend their understanding to solve another kind of questions or there is variation of questions. However, due to time constraint and the pupils' ability, they found it difficult to integrate much mathematical thinking in the lesson.

Nevertheless, when challenged to suggest other possible ways of integrating mathematical thinking in this lesson, they suggested the best way as asking a lot of "why" questions. For examples:

"why must be divided by 100 to get the percentage?"

"why converting from fraction to percentage, we use multiplication? But converting from percentage to fraction, we use division?"

"Why do we need to score full mark to be the winner?"

Another suggestion was encouraging pupils to give alternative methods of solving.

Teachers' Reflection on Lesson 2

Teacher who taught lesson 2 was Ms M. On reflection, she acknowledged that she did not follow the lesson plan strictly. She did not manage to cover all parts of the lesson because she believes that, "if pupils could not understand, there is not point to go on." Due to short of time, she changed the last part of the lesson to ask pupils to continue the following day. For her, today's lesson was not of any special but as what she normally did in class. However, her colleagues who were observing Lesson 2 felt that the class atmosphere was very lively and pupils seemed to enjoy the activity. All teachers were amazed with the number of TV programme and the familiarity of the pupils about these programmes.

Mr L observed that some pupils were able to explain the alternative methods that they suggested, this shows that they were thinking mathematically. He found some pupils were arguing among themselves when they were doing the matching activity. Some pupils used trial and error, some started to write down and calculate. Most pupils seemed engaged and enjoyed themselves. Ms K and Ms S echoed that they were a bit worried that the pupils could complete the matching activity successfully. This was because they have attempted to solve the problems while preparing the activity. It took them quite some times to find a match for one of the questions. They were very happy to see that all pupils could find the answers correctly.

Ms C gave some suggestions for improving the teaching such as pasting the questions on the board so that every pupil can refer to the question. She also suggested that besides the multiple additions (30 + 30 + …) and multiplication (30x6), another way is grouping of 30 + 30 become 1 hour, so 3 groups of 30+30 become 3 hours.

All the teachers agreed that although Lesson 2 appeared simple and easy, teacher Ms M has managed to incorporate mathematical thinking in the lesson. The teacher has asked a lot of "why" questions and has always encouraged pupils to suggest alternative methods of solving. She also encouraged pupils to present their solutions in front of the class.

Teachers' Reflection on Lesson Study

All teachers agreed that participating in lesson study gained them a lot of new ideas and new experiences. They felt better collegial collaboration with their colleagues. However, in spite of the benefits, they felt lesson study was a challenging task. They lamented that each lesson plan using the lesson study cycle required at least 3 to 4 weeks to be completed. In view of the present school system, they were overloaded with tons of paper works besides teaching load. They were over-stressed and rather reluctant to continue lesson study process. These grouses were also reflected in their journal writing. This shows that "time" remains the biggest challenge to the sustainability of lesson study process.

Teachers' Reflection on mathematical thinking

After the two cycles of lesson study, I discussed with the teachers about their understanding and importance of mathematical thinking. The school principal also joined us for the teaching observation of Lesson 2 and the reflection and discussion after that.

Ms C commented that she used to promote mathematical thinking in her normal class, such as variation in difficulty level (from easy to difficult), variation in types of question and variation in methods. However, she was not aware about the term, mathematical thinking. She believes that it is pertinent to encourage pupils to think mathematically. Mathematical thinking should be an important part of mathematics learning.

Mr L supported Ms C's comments about the importance of mathematical thinking. He remarked that mathematics lessons that involve activities that promote mathematical thinking appear more lively and enjoyable. By encouraging pupils to use various kinds of methods will make them more flexible in thinking. This might enhance their adaptability to daily life and future career. All the other teachers also agreed that the normal mathematics lessons are usually very boring and inflexible [死扳]. Pupils are usually asked to follow exactly what the teacher taught. Hence, mathematics lessons should include activities that promote mathematical thinking. The school principal especially agreed that it will be ideal if every mathematics lesson can help to develop pupils' mathematical thinking.

However, time remains the biggest challenge for these teachers. They lamented there were too much workload and documents that they have to prepare daily. Mathematics lessons that promote mathematical thinking usually take time to prepare and to engage pupils to participate. In addition, with the present school system that emphasis on examination, teachers and pupils are forced rushing to finish the syllabus, and to ensure pupils are prepared for examinations. Hence, it is too challenging and stressful to incorporate mathematical thinking in every mathematics lesson unless there is reform in the present school system, examination culture and emphasis of mathematical thinking.

Conclusion

This paper reported an exploratory study that aimed to promote mathematical thinking among pupils of a Chinese primary school in Malaysia. Even though all the five mathematics teachers participated in this study were familiar with lesson study process, they were not clear how to help pupils develop mathematical thinking. For these teachers, several ways of promoting mathematical thinking are (i) ask a lot of "why" questions; (ii) encourage alternative methods of solving; (iii) variation in types of question, so that pupils are encouraged to extend their knowledge to apply to new areas.

These teachers agreed to the importance of mathematical thinking and were keen to promote it. However, they remain sceptical about the practicality and feasibility of this project. They lamented the biggest challenge is time factor. They consider a mathematics lesson that promotes mathematical thinking to be always time consuming and effort driven. Nowadays all teachers are overloaded with both teaching and non-teaching duties. They are always stressed by the school authority and pupils' parents to complete syllabus in time and to ensure their pupils excel in public examinations.

In brief, the experience of this exploratory study implies that it remains a big challenge to promote mathematical thinking in Malaysian schools. Several hindrance are (i) school culture; (ii) teachers' attitude and commitment; (iii) teachers' workload; (iv) exam-oriented culture and (v) assessment system. Unless there are efforts to reduce these hindrance, or else the road to promote mathematical thinking in Malaysian mathematics classroom seems to be still far-fetched.

Acknowledgement

This study was made possible with the cooperation, sacrifice in time and effort of Ms Goh Siew Ching and her school principal and colleagues, as well as the pupils of the two classes. Special thanks also to my two postgraduate students, Hwa Tee Yong and Gan We Ling in helping to video tape the lessons.

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Appendix I: **Lesson Plan 1**

Subject	:	Mathematics
Year	:	Year 5
Learning area	:	Percentage
Sub-topic	:	(a) Convert proper fractions to percentages. (b) Convert percentages to fractions.
Duration	:	60 minutes
Resources	:	Blackboard, manila cards, marker pens, cardboards, hundred square paper, LCD, laptop
Key words	:	Percentage, symbol, percent, hundredths, hundreds squares, parts, convert, fraction, denominator, numerator, equivalent, simplest form.
Learning Objective	:	Pupils will be able to understand and use percentage.
Learning Outcomes	:	Pupils will be able to 1. Convert proper fractions to percentages. 2. Convert percentages to fractions in its simplest form.
Previous knowledge:		Pupils have already learnt the name and the symbol for percentage.
Values	:	Self-reliance, logical thinking, mathematical thinking, cooperative, bravery, gratitude, careful, helpful.

Appendix II: **Lesson Plan 2**

Subject	:	Mathematics
Year	:	Year 4
Learning area	:	Unit 5 Time
Sub-topic	:	Basic operations involving with time: Add minutes with answers in hours and minutes
Duration	:	60 minutes
Resources	:	Blackboard, manila cards, marker pens, cardboards

Key words : Convert, relationship, involving

Learning Objective : Pupils will be able to do the basic operation involving time.

Learning Outcomes : Pupils will be able to
3. Add minutes with answers in hours and minutes.
4. Convert units of time involving hours and minutes.

Previous knowledge: Pupils have already learnt time in hours and minutes and converting units of time involving hours and minutes.

Values : Logical thinking, mathematical thinking, cooperative, bravery, honesty, careful, helpful.

Step	Content	Activities		Remarks
		Teacher	Student	
Set induction (± 1 minutes)	Asking questions related to their daily life.	T: What is your favourite TV programme?	Various answers will be given by the pupils .	Pupils listen and respond.
Development 1 (± 9min)	Variation of questions .	Teacher asks the following questions and led the pupils to answer. T: So, how many minutes you spend to watch your favourite programmes on Monday? T: Tuesday? Wednesday?..... T: How much time do you spend on watching TV programs in a week? Teacher will draw a table concerned on	Various answers will be given by the pupils Pupils find the duration of the time spent for TV programmes on each day and the total time spend in a week.	Pupils listen and respond.

		the blackboard and asks the pupils to find the duration of the time spent on TV programmes each day and the total of time spent in a week. Teacher also remind the pupils the moral value behind it, i.e. don't spend too much time on TV programmes, but instead have to choose the good programmes.		
Development 2 (± 46min)	Jigsaw puzzle: two sets of questions i) Easier Questions: Purple, pink, Green cards. (20 minutes) ii) Difficult Questions : Blue, yellow, orange cards (20 minutes)	1st round: Easier 1. Divide the pupils into 6 groups. Each group consists of 4 or 5 pupils. 2. One representative of each group comes forward to get an envelope. 3. Inside each envelope, there are 2 pairs of questions. 4. Every pupil in each group think, discuss and to match the correct pairs of questions and answers so as to finish the task. 5. Teacher will then ask the pupils to come out to explain how they get the answers and discusses with the pupils.	Pupils cooperate to find the correct pairs of questions and answer and then paste the answer on a manila card in each group. Pupils present their 'works' on the blackboard. Pupils come out to explain how they get their answers.	Pupils discuss and solve the problems.

		2nd round: Difficult 1. Repeat steps 1to 5 as in 1st round.		
Closure (± 4 min)	Conclusion Enrichment	Teacher concludes the lesson. Every pupil will be given a copy of worksheet as homework for enrichment.	Pupils listen and solve the problems.	Worksheets