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-Focusing on Mathematical Communication-
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WE SHALL OVERCOME DYSFUNCTIONAL BELIEFS FOR INTRODUCING COMMUNICATION STUDY

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Sheet 1

I have the great pleasure to be here with you. My name is Hideyo Emori. I come from Gunma University, Japan. Personally speaking, this is the third time to come and give lectures at Khon Kane University. I am also happy to meet many Thai friends here again. I have been studying communication in learning mathematics for the last twenty years. When I started to study mathematical communication, at that time, I believed almost all the teachers agreed with the importance of communication in learning mathematics. And it was truly "Yes". None denied the importance of communication. But, many teachers hesitated to introduce communication activity in their classes, because they did not want kill the valuable time by introducing the activity of communication. The first conflict what I confronted with was how I should overcome the dysfunctional beliefs for introducing communication study. So, today I want to speak to you how I could overcome the dysfunctional beliefs for introducing communication study in Japan. And I also want to show you the new perspective concerning mathematical communication.

Sheet 2

Just now I mentioned, I suppose you may not deny the importance of communication. On the other hand, many teachers, who come and join in this conference today, feel some difficulties to introduce the communication activity rather than the ordinary lesson style. Then I want to pose a question as the introduction of my speech to you. The question is, "Do we hesitate to introduce communication in learning mathematics? Yes, we do, because we love the classic

style of teaching!"

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Twenty years ago, when I started to do the communication study in Japan, there was no mathematics educator who majored in this field. I was confronted with several dysfunctional beliefs. For example, many Japanese researchers asked me the reason why you selected such an uncertain research problem. And others asked me how the communication study connected with the researches of mathematics education. If you want to introduce the communication study as mathematics education into your country or region, you need to answer the question; "How dose the communication study connect with the researches of mathematics education?" If we can answer this question; then we can introduce communication in our math classes with confidence.

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If we can not tell teachers the importance of "Communication" in learning mathematics, teachers do not want to introduce communication in their classes, because they are still familiar with classic style of teaching.

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Generally speaking, if we have to change our life styles, then we feel many difficulties. This is also true for our style of teaching. If some teachers are familiar with the teaching style like that a teacher teaches and students listen. Then, they would say; "Why do we need to introduce communication in our classes? Is that really effect? This difference between such teachers and us is not so narrow. This is the big difference, because we have not been able to share the common value for mathematics education.

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Some teachers still believe; getting a right answer is important for solving a problem; working alone is the best way to improve one's competence.

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Before encouraging teachers to introduce communication in the class, we need to overcome these dysfunctional beliefs.

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Twenty years ago, I could not answer the basic question why communication is so important. I felt there were some dysfunctional beliefs behind such a question. For examples, “Only right answers count”. “Teacher tells you how to get those answers”. “Working alone is the best way to improve one’s competence”. “We can easily convey our thoughts to the others”. “Communication looks like playing catch with information among people”.

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On the other hand, I believe there were some functional beliefs for introducing the communication study. For examples, “The process to give birth a good idea counts rather than the product itself”, “Working together is the best way to improve one’s competence”, “Students talk with freshly fashioned ideas by using their own words”.

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“It is impossible to communicate our thoughts directly to the others.” “Communication is a complicated and complex phenomenon, but it is indispensable for mathematics education.”

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How should we change our beliefs? To answer the question,

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We need to know the roles of students as sense-makers, teachers as story-makers, researchers as value-makers of communication.

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First of all, we shall focus on the students as sense makers. If we recognize students as sense-makers, we must value students’ monologue, utterance, action, because these are the windows into the process of how our students construct the meaning for what they are learning.

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As the 2nd view point, we focus on teachers as story-makers. If we recognize teachers as story-makers, we must value teachers’ efforts to build up their classes in mathematics as a chained story, because this chained story what a teacher and

students make together influences to improve students' understandings as a whole.

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As the 3rd view point, we focus on researchers as value-makers. If we recognize researchers as value-makers, we must value the communication in learning mathematics and inform teachers how their teaching affects the process of students' thoughts.

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As the chapter 2, I want to focus on the necessity of changing the meaning of communication. Lesson study shows us the necessity of changing the meaning of communication in learning mathematics from the narrow sense to the broad sense.

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Usually, we believe communication consists of speaking, listening, writing, and reading. This kind of perspective of communication is too narrow. If communication only consists of speaking, listening, writing, and reading, then we will accept the narrow sense of communication. We will separate human thoughts from communication. If we stand on this narrow sense of communication, we may have a belief "What makes mathematical communication mathematical just depends on the way or method how we communicate. Mathematical communication means the communication what we use mathematical representations. But, I strongly believe this is not true. For example, we can easily have an image what two mathematicians talk about mathematics. They attach importance of their own ideas, they never concern about their expressions in their conversation. Perhaps, they enjoy omitting common knowledge and some information. In the case of such a simple communication, they can use some kind of abbreviations, because they can understand what their partner said by complementing each other.

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Therefore, we need to have another model of mathematical communication. This is the model of mathematical communication in the broad sense. In this model, communication consists of problem solving, reasoning, connecting, and communication in the narrow sense. I believe mathematical communication should be treated as the integrated activities in learning mathematics.

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When we discuss the process of learning mathematics, we never separate between communication and human thoughts. All experience in learning mathematics is carried out through communication. We sometimes concentrate our own thoughts by cutting any relation with others. On the other hand, we try to have something to do with others, if we suppose that we need to talk about what we thought. It just looks like that we put a switch of the communication with others, on or off. Even if some communication stops superficially, we had better consider its communication keeps on.

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If we change the perspective of mathematical communication from the narrow sense to the broad sense, we could integrate some researches of mathematics education. We could integrate researches of "Understanding", "Social interaction", and "Affection", into the communication study. The perspective of the classical research of understanding mathematics in the 1970s just focused on the personal and static understanding. In the 1980s, mathematics educators were interested in the role of "Social Interaction" and "Affection" when they discussed understanding. And in the 1990s, mathematics educators realized the importance of building up the dynamic model of understanding. Mathematics educators became aware of that the model of understanding should be treated within the social context. A student is not an isolated existence; he or she is a social existence. We cannot help communicating with others. We may try to solve a problem alone, but sometimes we want to cooperate to solve the problem. We have realized the model of understanding should be integrated into the model of communication.

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Now we can realize the communication study could be the ground research field of mathematics education. It means that communication study covers the cognitive process of our thoughts, the affective process of learning mathematics, the roles of social interaction in learning mathematics, and so on. This image of pyramid model shows us the role of communication study in the researches of mathematics education. Therefore we try to build up our own communication theory as the ground theory of mathematic education by ourselves. Twenty years ago, I could not answer the question; "How dose the communication study connect with the researches of mathematics education?" But now, I want to answer to this question

like this. The communication study should be the ground theory of mathematics education.

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We already realized the meaning of broad sense of communication. Communication connects problem solving, reasoning, and interpreting messages. This perspective of mathematical communication is so important. Then I try to show you how mathematical communication connects between problem solving, reasoning, and interpreting messages with showing you some examples.

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This is an example for how we need to solve a problem to interpret mathematical message. An announcer said, “The economic growth rate of the first quarter of this year is 2%. And this rate becomes 8% as an annual growth rate”. We often hear or read this kind of news in our daily life. And if we want to understand what this announcer said, we try to solve this problem. “How do we calculate the annual rate?” I suppose there are many people who calculate “2 times 4 is 8”.

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But, in fact, this calculation “2 times 4 is 8” is not right. [*Animation 1] Within the mathematical communication, it is not enough for us to interpret the message what we receive literally. Until now many researchers think of this example as a model of mathematical communication. The statement of this news includes a way of mathematical thought. And it consists of a mathematical notation such as a sign "%" to express the economic growth in a ratio. However, most of the people who feel it mathematical, misunderstand the mathematical structure of this message. They think this statement implies a multiplication “2 times 4 is 8”. If receivers interpret this statement like this, this communication betrays the rigorousness of the logic of mathematics. [*Animation 2] The statement of this announcer connoted the structure premised by the calculation of $(1.02)^4$, 1.02 to the power four, is 1.08243216. This is the right interpretation. You may understand that it is not enough to interpret some message literally, when we want to participate into the mathematical communication. We sometime need to solve the problem to understand the communication itself, when we want to take part in the mathematical communication.

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This is the second example for how mathematical communication emerges the new idea. And here, “to emerge the new ideas” means “to create the complete new ideas”. Before showing you the second example, I shall read the problem which three Japanese 5th grade students talked about in their math class. “There are twenty houses. If we draw a telephone line between every two-house, how many telephone lines do we need”?

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Three Japanese 5th grade students talked about how to draw the lines in the case of 3 houses. Student A wrote two houses and drew one line between them. [*Animation 1] And finally, Student A wrote the 3rd house [*Animation 2], and drew one line [*Animation 3], and said, “I can't understand how to draw the lines”.

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And then, Student B gave a quick response to student A. Student B said, we have to connect every two-house with one line, and therefore we have to connect between these two houses. [*Animation 1] Student B drew the green line to connect two outside houses.

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After Student B’s feedback to Student A, Student C said quickly, “It looks strange! So, I think it is better to move this mark here like this.”Student C covered two lines and one house. [*Animation 1] And Student C move the right side house below the two houses [*Animation 2, 3], and drew two lines to connect houses. [*Animation 4, 5] Finally Student C showed his friends a model like a triangle. To this triangle model, almost all students in this class applauded Student C and some of them clapped their hands.

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In the Communication among these three students, Student A said, “I can’t understand how to draw the lines” at first. Student B received the message sent by Student A [*Animation 1], and then Student B replied to Student A. Student B sent a feedback to Student A. [*Animation 2] And later, Student C quickly replied to Student B. [*Animation 3, 4, 5] This feedback also sent to Student A as the chained feedback to Student A. [*Animation 6]

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After these conversations, students tried to solve the problem in the case of twenty houses by themselves. They never exchanged their ideas in the least for fifteen minutes. Then I focused how Student A, B, C solved this problem, because I wanted to know the effect of this communication among three students. You may know Student C was applauded by his classmates. Student B also taught the meaning of this problem to Student A. So we could imagine Student A had the least understanding at that time. We never thought the way of the problem solving by Student A was the most excellent in her class. Student A caught plenty of information from her own original interpretation of messages of Student B and Student C. Student A solves this problem by calculating the sum of 1+2+3+…..+17+18+19. We found Student A interpreted Student C's message like this figure. [*Animation 1, 2, 3, 4, 5] In the case of three houses, Student A took the selective perception for the figure of Student C as one line plus two lines.

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From this interpretation, Student A built up the way how to connect the lines in the case of 4 houses as 1 + 2 + 3. In the case of two houses, we need one line. In the case of three houses, [*Animation 1, 2, 3, 4,] we add to write two lines like this. And in the case of four houses, we add three more lines like this. [*Animation 5, 6, 7, 8, 9]

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Student A connects between interpreting the figure of Student C, problem solving, and reasoning. I think this is an authentic example for emerging a new idea.

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This case study indicates us the potentialities that a receiver will be able to give birth to richer information rather than that of sender's intention by decoding message.

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We define this type of communication chain "emergent chain". In the emergent chain, a sender stimulates receiver's thought. The receiver links the message to his already stored pieces of knowledge, and reconstructs his knowledge. This reconstruction of knowledge results in creating a new idea.

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Now we concerned about the relation between problem solving, reasoning, interpreting messages. So let's show you another connection briefly. As the 1st example, I would like to show you that communication connects representation and reflective thinking. Consciousness which is brought out thorough the activity of communication in leaning mathematics connects representation and reflective thinking.

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The 2nd example is that communication connects constructing concepts or ideas and mathematical activities.

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The 3rd example, communication also connects cognitive experiences and affective experiences, or emotional experiences.

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The 4th example, communication connects personal isolated thoughts and others' thoughts.

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As the concluding remarks of chapter 2, we can say that "Communication" is one of the most important keyword for unifying all activity in learning mathematics.

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As the chapter 3, I will show you a framework of the research of communication for cognitive development. This is a basic model for the research of mathematical communication.

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There are many elements in the process of communication, but I believe "sender", "message", and "receiver", these three elements are the most basic elements. Then we start the communication study by focusing on these three elements. And especially, I believe the concept of message is the most important concept for the communication study. And when we discuss the concept of message, we also attach importance to the concepts of "Process" and "Interpretation".

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These are four key concepts for communication study; “Message”, “Process”, “Interpretation”, and “Feedback”.

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Now I tell you the concept of message is the most important concept for the communication study. Message is a physical product made by sender, which has no meaning. Communication is not the transmission of meaning. It is impossible to transmit a meaning.

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As the 2nd key concept, we had better to focus on the concept of “Process”. Communication is not the fixed phenomenon. All elements included in the communication process act mutually and affect each other.

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And the 3rd key concept is the concept of “Interpretation”. We interpret a message to bring out a meaning as which a sender intend to send some information to a receiver. That is a reason why one message brings several kinds of effect to different receivers.

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Now we are talking about three elements and three key concepts for communication study. As I told you so far, the process of communication is not the process of sender sends a message to a receiver. I mean the process of communication does not end in the one way. In the usual communication, the receiver gives some message to the sender as feedback. If we will add the 4th concept “feedback”, we can show you a model of a basic cycle of “Social Interaction”. A basic cycle of social interaction consists of a transmission of the first message and its feedback.

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Please pay attention to my explanation for these key concepts. The most important principle of communication study is the definition of message. Please pay attention to the definition of message, again. “The message is just only a physical object.” That is the reason why the transmission of a message dose not mean the direct transmission of information. Receiver’s subjective interpretation produces a

meaning, because the interpretation depends on the person. Then we use the word of “subjective”. The interpretation is basically subjective interpretation. When we analyze the communication, we should try to know the difference of subjective interpretations among students.

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This basic principle leads us the statement below. One message acts on receivers positively or negatively, and its effect depends on the receiver’s subjective interpretation.

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As we know, we have to unify all mathematical activity into the communication. Therefore we need to introduce communication study in our country!

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If there are some dysfunctional beliefs for introducing communication study, I strongly believe they are brought by one unguided and thoughtless belief; “Message conveys a meaning”.

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If we want to overcome this dysfunctional belief for introducing communication study, we have to start our research from the basic principle; “Message is just only a physical object.”

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Thank you so much. Shall we start Communication study with keeping our own ethnic identity, because communication depends on the language we speak and the culture we live in? If the communication study could be the ground theory of mathematics education, we need to build up our own communication theory in learning mathematics.

Finally, I also appreciate Dr. Maitree Inprasitha who is the organizer of this conference, and all of the members of the organizing committee of this conference. And special thanks to Thai Students of CRME, Khon Kaen University. Thank you so much again.

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