

TEACHING AND LEARNING MATHEMATICS THROUGH LESSON STUDY – AN EXAMPLE FROM HONG KONG¹

Frederick K.S. Leung,
The University of Hong Kong
Yuk Ying Yuen, CCC Hoh Fuk Tong College. Hong Kong

Introduction

In recent years, continuing professional development of teachers has been strongly advocated in Hong Kong (ACTEQ, 2003), and one of the means for professional development of the teachers is through lesson study, where teachers of the same school work together to study their own lessons for the improvement of classroom teaching (Fernandez and Yoshida, 2004). Lesson study involves teachers preparing lessons together, observing and evaluating each others' lessons, and having discussions throughout the whole process. They share their teaching experiences and form a supportive group and review their classroom practice regularly.

In this paper, an example of a lesson study in Hong Kong in the subject area of mathematics will be reported. The level of students involved is grade 10, and the topic chosen for study is: *To enhance the teaching and learning of Mathematics ---“Solving Simultaneous Equations by Graphical Method”*.

In the following sections, lesson study as understood by teachers involved in this project will be discussed. This is followed by a description of the background of the study. The procedures for conducting the lesson study will then be described in detail, and the results of the study will be presented. Finally, some reflections on the lesson study will then be made, and the limitations of the study will be pointed out.

What is lesson study?

Lesson study may be considered a kind of action research consisting of a spiral of steps involving planning, fact-finding and execution. The process may be perceived as

¹ Paper to be presented at the APEC International Symposium on Innovation and Good Practice for Teaching and Learning Mathematics through Lesson Study, 14-17 June 2006, Khon Kaen, Thailand.

an action-reflection cycle of planning, acting, observing and reflecting (McNiff, 2002; see Figure 1 below).

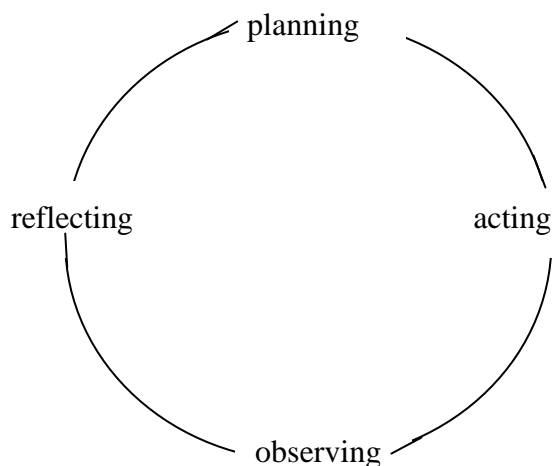


Figure1: Lesson study cycle (McNiff, 2002)

During each cycle, it is expected that small incremental improvements would be made, and as a result experiences of good practices can be accumulated and can be shared among teachers.

The first step of a lesson study is to identify a problem in teaching and learning, and this is usually achieved by a group of teachers gathering together and discussing the problems they have encountered in their past teaching experience. After identifying the problem, the group of teachers draws up a plan to collect the data they need to know about the present situation of their students, and the criteria of success for the study are established. Then the teachers prepare the lessons together, and one of the teachers teaches the lesson as planned. The teaching is observed by the rest of the teachers, and the lesson is usually videotaped. After that the teachers evaluate the lesson together to find ways to improve the lesson. The teaching plan is then modified, and the lesson is taught to another class using the refined lesson plan. Then the teachers gather together to evaluate the effects of the actions taken. Evidences are gathered to assess how far the criteria of success have been met. Finally the teachers reflect on the whole process together to identify ways the process has impacted on their work and their professional development. The impacts of the lesson study on the whole school, if any, are also discussed.

Background

The study took place in a medium sized secondary school (student population of about 1100) in a sub-urban area of Hong Kong. The school has been established for about forty-five years and is run by a Christian organization. Students are of average academic standard, and there is an emphasis on enhancement of teaching and learning by the school. The medium of instruction² of the school is Chinese. There is a culture of collaboration among teachers in the school, and continuing professional development of teachers is emphasized.

A group of four mathematics teachers, under the leadership of the co-author of this paper (who is also the panel chair of the mathematics department in the school), participated in the study. The four mathematics teachers (including the panel chair) involved in this project were all teaching different classes of Secondary 4 (i.e., grade 10), and the target group for study was the Secondary 4 students in the school. The aim of the study is to find ways to improve and enhance the teaching and learning of mathematics, leading to professional development of the teachers concerned. It is believed that the most effective way of improving teaching is for it to be done in the context of a lesson study. In this paper, lesson study of one lesson only will be reported.

Procedure

Timeline

In the first meeting of the project, the topic for study was decided based on an analysis of the current situation of the school (see below), and the plan and timeline for implementation were set. The timeline was established so that team members knew how the time would be spent at different stages of the project. The first meeting was held in November 2004. After negotiation among team members, it was agreed that the lesson study itself would be conducted in the last week of April 2005. This was because all team members had a lot of duties in the first term and extra time was needed to do the preparatory work. Two S.4 (i.e., grade 10) classes were chosen for this study. A pre-test would be given to the classes about one week

² In Hong Kong, about 3/4 of the secondary schools use Chinese, the mother tongue of the vast majority of the population, as the medium of instruction (MOI), and the MOI for the remaining schools is English.

before the lesson.

The second meeting was held in February 2005, and the main issue discussed in this meeting was about the lesson plan (see below). The third meeting was held in the first week of April, and work and duties were distributed among the teachers involved.

The fourth meeting was held immediately after the first study lesson had been conducted, and the main purpose of the meeting was to evaluate the first lesson and suggest modifications. The fifth meeting was held after the second trial, when evaluation and reflection for the whole project were conducted. The whole study finished by May 2005. A teaching assistant for mathematics in the school was responsible for preparing the videotapes for the lessons under study. The procedures above can be illustrated in Figure 2 below:

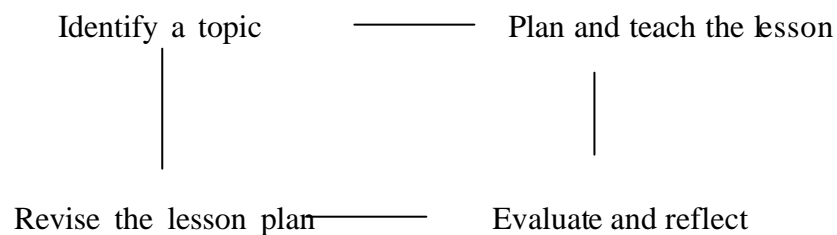


Figure 2: Procedure for conducting a lesson study

Analysis of the current situation

Before the lesson study, a holistic review of the current situation of mathematics teaching and learning was conducted. Facts on the current practices and ways of improving the practices were gathered. First the mathematics curriculum in the school was studied critically by the teachers who participated in this project to ensure that the requirements of the teaching syllabus were well understood by the participating teachers. Secondly, the study team wished to know more clearly about the standard of the students and the problems they faced in learning mathematics. Meetings were held to discuss which topic to choose for this lesson study in order to enhance the teaching and learning of mathematics. A number of mathematics topics have been considered for study, and after discussions among the team members, the

topic “*To improve students’ understanding of solving simultaneous equations by graphical method*” was chosen for study. The reason for choosing this topic was that the team found that most students were weak in plotting and reading graphs. They lacked practices in plotting graphs in their junior forms. It was hoped that through improved teaching, students’ ability in handling graphs would be improved, as graphical method is important in learning mathematics and other related subjects. Relevant teaching materials and documents such as the scheme of work and the textbook were then studied carefully by the team. A pre-test was given to the students before the lesson in order to get a clear picture about whether students had a good command of the prerequisite knowledge for that topic. In order to know more about the strengths and weaknesses of the students, teachers of junior forms were consulted too.

During the meetings at this stage, the team tried to be critical of their own teaching methods and the way their students learn, and several aspects of the weaknesses in classroom teaching and ways of implementing changes were identified. The team was very clear about the teaching goals and the learning targets, and teaching strategies were designed to help students attain those learning targets.

Preparation of the lesson plan

A lot of time was spent in discussing the lesson plan. The following points were considered while preparing the lesson plan:

- to understand the prerequisite knowledge of the students
- to set clearly the teaching goals of the lesson
- to find out the misconceptions or knowledge gaps of students in learning this topic
- the use of a suitable software and good design of worksheets to help the students understand the underlying mathematical concepts and skills
- the instructional strategies and the learning activities during the lesson, that is the interaction among the teacher and the students
- the time allocation for each learning activity and the ways to assess the understanding of the students.

After discussion, it was decided that students should know first how to solve two linear equations simultaneously by graph and then used a similar method to solve one linear equation and one quadratic equation graphically. The software “Sketchpad” would be used for illustration during the lesson. The final lesson plan can be found in Appendix 1.

Modifications of the lesson

After the lesson planning, one teacher did the teaching to one of the classes (S.4C) according to the lesson plan and the lesson was videotaped in order to find out how the students learned in the classroom and their responses to the teaching. After that lesson evaluation was carried out among team members and various ways of improving the teaching practices were discussed. For example, it was found that the original plan included too much content and some parts of the lesson had to be deleted. Also the effect of the use of IT was not so good, and some modifications were done. Then the lesson plan was revised and the lesson was taught by another teacher to another class (S.4B) about one week later using the refined lesson plan. This second lesson was also videotaped for the final evaluation and also for future professional development use. So there were altogether two cycles in this study.

Criteria of success

For a lesson study, it is important to establish the criteria of success so that one knows to what extent the project has succeeded. For this lesson study, the criteria of success are summarized by in able 1 below:

Table 1: Criteria of success for the lesson study

Criteria for success	Evidence gathered
Active participation of students	Active participation of the students in class, with good responses to teacher’s questions and activities were observed.
Better learning of students	The students showed improvement when comparing the result of the pre-test with that of the post-test.

Improved instructional practices of teachers, and gain in professional development	From the comments of the report on the lesson observation, the feedback from colleagues was very positive. The flow of the lessons was smooth.
Improved management skills of teachers	Feedback from colleagues on the project as a whole was encouraging. Team members were willing to try the lesson study again in the next academic year.

Collaboration among team members

As pointed out above, there were four teachers involved in the project, and each of them was responsible for different parts of the project. One teacher was responsible for writing the lesson plan after discussion of the whole team. Two teachers prepared for and did the teaching. And one teacher was responsible for designing the tests, the worksheets (see Appendices 2 and 3) and the IT teaching aids. The minutes of the meetings were taken in turn by the four teachers. A teacher assistant was responsible for making the videotapes. So it was a truly collaborative project.

The four teachers in the project worked as a team, and every team member's ideas were respected and their opinions treasured. Team members worked collaboratively and all were empowered to do the project. All the teaching materials such as the teaching plan for the lesson, the teaching aids, the worksheets, the pre-test and the post-test, etc. were prepared by the team collaboratively.

Support from the school

In the school, this lesson study project was highly supported by the principal. Actually, a whole school approach was adopted. Besides Mathematics, lesson study was also conducted in the subjects of English and Chinese. All S.1 to S.4 (i.e., grades 7 to 10) teachers of these three "core" subjects were involved in lesson study, and a common free period was scheduled every week so that colleagues could meet

and discuss about the lesson study. At the end of the term, a sharing session on these lesson studies was organized by teachers of the three subjects for all the teachers in the school. This led to both school improvement and enhancement of teachers' profession, and a win-win situation was achieved.

Results

Scores of the pre-test and post-test

As mentioned above, a pre-test and a post-test (these two tests were identical) were administered to each of the two classes under study, and the scores of the tests are shown in Tables 2 and 3 below:

Table 2: Scores for class S.4C (lesson taught based on the original lesson plan):

Marks	Pre-test (No. of students)	Post-test (No. of students)
0-19	3	1
20-39	7	1
40-59	22	23
60 or above	4	11
Total no. of students	36	36
Full marks	64	64
Mean marks	46.7	55.8
Standard deviation	15.0	10.3
Maximum mark	64	64
Minimum mark	6	6

Table 3: Scores for class S.4B (lesson taught based on the revised lesson plan):

Marks	Pre-test (No. of students)	Post-test (No. of students)
0-19	0	0
20-39	6	2
40-59	23	19
60 or above	12	20
Total no. of students	41	41
Full marks	64	64

Mean marks	51.7	55.4
Standard deviation	10.1	12.0
Maximum mark	64	64
Minimum mark	29	31

As shown in the above tables, both classes gained good improvements after the lesson. The number of students who obtained a mark of 60 or above was significantly increased.

Evaluations by team members

Besides students showing improvements as shown by the results of the pre -test and post-test scores, below are the results of the project based on the evaluations by the team members:

- The lessons run smoothly. Most of the students concentrated in the lesson and participated actively in class. Some students who did not use to participate in class even asked questions during the lessons.
- Students could find out the answers from the graphs but they did not know how to write the answers correctly. This was revised in the second lesson.
- There were too much teaching contents to be covered. This was revised in the second lesson too.
- There should be more examples in the worksheets so that the students could follow the examples and complete the worksheet.
- More discussion could be given to the students. Interactions among the students should be encouraged.
- After the first trial, it would be better if more time was allowed to reflect and evaluate on the lesson before doing the next one. There was a lack of time to go through all the teaching materials by all teachers before the lessons.

Student responses

Some of the students were interviewed informally after the lesson and their opinions on the lessons were gathered. Most of them felt that they understood the lesson and were able to complete the worksheets given to them during the lesson. They enjoyed the lesson. So the above criteria for success were all met to a certain extent. Of course there would always be room for improvement in a lesson study.

Reflections on the lesson study

It was felt that in general the lesson study has positive effects on students, teachers, and the school as a whole.

Effects on the students and teachers

- The lesson study led to direct improvement of teaching and learning.
- The teaching materials were suitable for students in this year but might not be suitable for use again next year. However teachers' experiences and involvements in the project were valuable and could be applied to other lessons.
- There was good collaboration among the teachers in the school on this project. Team members all had equal power to discuss, and to criticize the ways of conducting the lesson. They learned more about different teaching methods from each other. Through negotiations, team members learned how to understand the perceptions of others and worked collaboratively for a good lesson.
- Evaluating and reflecting upon the lesson were very important. These might lead to improvement in the next cycle. During the process, adjustments of the lesson plan could be made from time to time.
- Teachers who took part in the lesson study would see themselves making contributions to the development of knowledge and teaching profession. This kind of classroom research could improve the teaching and help teachers in their professional development.
- Managerial skills of team members, especial those of the panel head, were improved. Members learned to share their points of views and their own teaching practices. They also learned to be respectful to others in decision making.
- Enthusiasm of the teachers towards teaching was improved, and this was important in the way to succeed.

Effects on the school

At the end of the term, there was a sharing session arranged by the school for all staff. Team members shared their experiences and their reflections on the application of the lesson study to teaching and learning. There was a great impact on the teachers.

As a result, it was decided that more collaborative teaching would be done in the coming year.

Limitations of the project

Though there are many advantages in lesson study, it needs a lot of time and resources doing it. In Hong Kong the work load of teachers is so heavy that this kind of lesson study cannot be done often. Scheduling of time-table so that teachers have free common time-slots for conducting lesson study is a major challenge. Recording the lessons for later discussion is one way of meeting this challenge, but the most severe problem is that teachers do not have much time to have discussions.

Notwithstanding these limitations, from the view of professional development of teachers, lesson study is still considered worth doing.

Concluding remarks

Since the society and the world are changing rapidly, we need to introduce new ideas for teaching and learning from time to time. Lesson study is very suitable to be used as a tool to fulfill this need. This research-development system is worth trying in schools. Lesson study is a self-evaluation and self-correction process. The students, the teachers and the school all gain benefits from it.

References

- Advisory Committee on Teacher Education and Qualifications (2003). *Towards a Learning Profession: The teacher competencies framework and the continuing professional development of teachers*. Hong Kong
- Fernandez, C. and Yoshida, M. (2004). *Lesson Study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, N.J. : Lawrence Erlbaum Associates.
- McNiff, J., Lomax, P. & Whitehead, J. (1996). *You and Your Action Research Project*. London: Hyde Publications (pp 29-45).

Appendix 1				
Unit	Graphical Solutions of Simultaneous Equations			
Reference of teaching materials	“New Progress in Certificate Mathematics” Hong Kong Educational publishing Co.			
Teachers	MFC SKP	Classes taught	S.4	No. of Students: 36(4C), 41(4B)
Dates of teaching	21-4-2005 (MFC S.4C) 27-4-2005 (SKP S.4B)		Teaching period	1 period (40 mins.)
Students’ Prerequisite Knowledge	<ol style="list-style-type: none"> 1. Understand how to plot points in a coordinate plane 2. Know how to plot a linear equation on a graph paper 3. Understand that the graph of a linear equation in two variables is a straight line 4. Understand that all the points on the straight line can satisfy the linear equation 5. Know how to use graphical method to solve a system of two linear equations 6. Understand that the solutions found from the graph are only approximate solutions 7. Understand that the point of intersection of two straight lines can satisfy both equations and hence it is the solution of the simultaneous 			

Teaching Goals	<ol style="list-style-type: none"> 1. Students are able to use graphical method to solve a system of two linear equations of two variables.(as a revision) 2. Students are able to know that the solutions found from the graph are only approximate solutions.(as a revision) 3. Students are able to draw a straight line by plotting three points 4. Students are able to understand how to find the solution of a system of equations, one linear and one quadratic graphically and its meaning. 5. Students are able to read and write the solutions from different scales of graphs.
-----------------------	---

Preparations before the lesson:

1. The software “Sketchpad” will be used to prepare the graphs for illustration in the lesson.
2. The worksheets will be prepared. All the quadratic graphs will be provided and the students need to draw straight line graphs only

Teaching aids: Computer for demonstration and the software “Sketchpad”

Teacher’s Activities	Students’ Activities	Assessments and Points to be noted
<p>Explain briefly about the learning goals of the lesson</p> <p>Explain how to use graphical method to solve a system of two linear equations of two variables using Sketchpad. Ask students to find the solution by themselves and write it down on the worksheet.</p> <p>Discuss with the students how to find out the answer and explain why the point of intersection of two linear equations is the solution.</p> <p>Ask the students to check the answer.</p>	<p>Listen</p> <p>Students try to find the solution And write it down.</p> <p>Students discuss in groups and share their opinions.</p> <p>Do the computation by the aid of calculator.</p>	<p>Time:15 mins.</p> <p>Check that students can find the point of intersection of the two lines and write the correct solution.</p> <p>the point of intersection of the two lines can satisfy both equations</p> <p>By checking students are able to understand that the answer is an approximate solution only</p>

<p>Tell the students first that the graph of a quadratic equation is a parabola. Give an example: How to find the solution of a system of equations, one linear and one quadratic graphically. Given a graph of a quadratic Equation. Ask the students to complete the table given and use the three points to plot a straight line.</p> <p>Using the graph plotted by Sketchpad Discuss with the students how to find the solution of a system of equations, one linear and one quadratic .</p> <p>7. Ask the students to do more examples, using graphs of different scales.</p> <p>8. Ask the students what is the maximum number of solutions for simultaneous equations, one linear and the other quadratic.</p> <p>9. Ask the students the reason why the point of intersection of the graphs is the solution.</p>	<p>Do the computation and draw graph.</p> <p>Discuss in groups, find the solution and write it on the worksheet.</p> <p>Practice more to find the solutions from the graphs</p> <p>Students answer “Two”</p> <p>Group Discussion</p>	<p>Time: 20 minutes</p> <p>See if the students can draw the graph correctly.</p> <p>See if the students are aware that the solution is an approximate solution.</p> <p>See if the students know there are different cases for the number of solutions</p> <p>The solution is only an approximate value by checking. The point of intersection can satisfy both equations. Hence it is the solution of the simultaneous equations</p>
<p>10. Conclusion</p> <p>(a) We can find the solution of a system of equations, one linear and one quadratic graphically.</p> <p>(b) The solutions obtained from the graphs are only approximate values</p> <p>(c) The point of intersection can satisfy both equations. Hence it is the solution of the simultaneous equations</p> <p>11. Assignment given</p>	<p>Listen</p>	<p>Time:5 mins</p>

Appendix 2

Form 4 Mathematics Solving Simultaneous Equations by Graphical Method (Worksheets)

Name? _____ Class? _____ Class Number? _____

1. Solve the following simultaneous equations by graphical method.

$$\begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

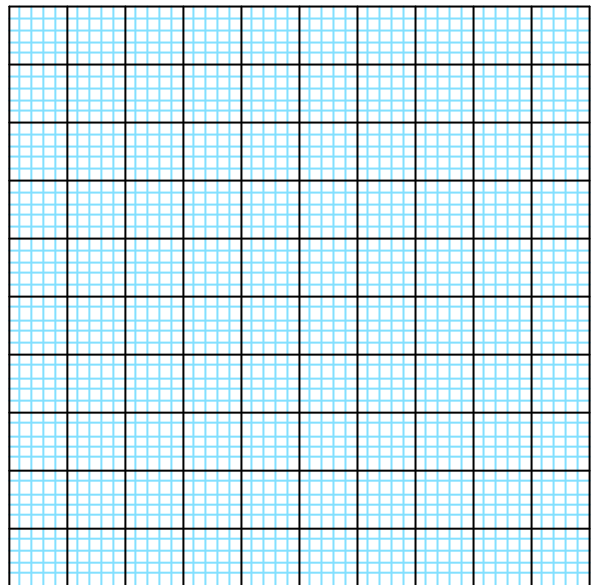
Solution

$$x + y = 4$$

x	0	2	4
y			

$$x - y = 2$$

x			
y			



∴ From the figure, the point of intersection is _____

∴ The solution of these simultaneous equations is _____

Checking?

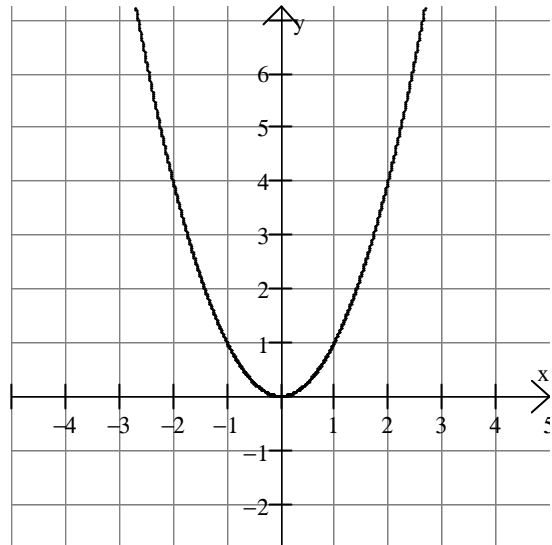
- 2 Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ y = -x + 2 \end{cases}$$

Solution

$$y = -x + 2$$

x	-3	0	3
y			



∴ From the figure, the point of intersection is _____

∴ The solution of these simultaneous equations is _____

3. Textbook p.57 (Follow-up Exercise)

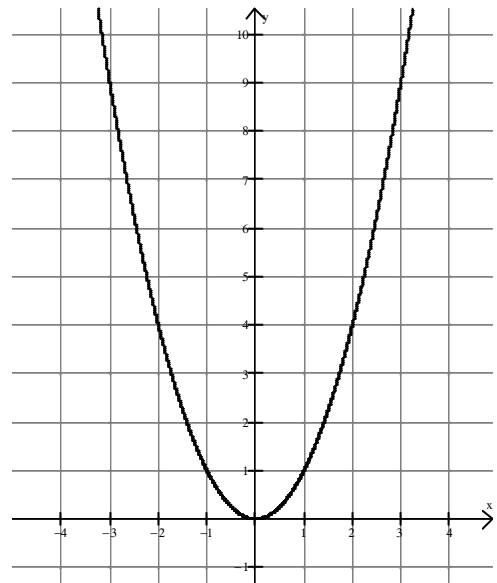
4. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ y = 6 - x \end{cases}$$

Solution:

$$y = 6 - x$$

x			
y			

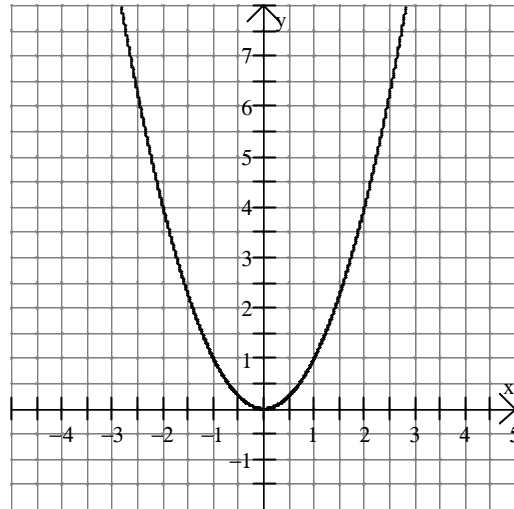


∴ From the figure, the point of intersection is _____

∴ The solution of these simultaneous equations is _____

4. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ 2y = 3x + 2 \end{cases}$$



Solution

$$2y = 3x + 2$$

x			
y			

∴ From the figure, the point of intersection is _____

∴ The solution of these simultaneous equations is _____

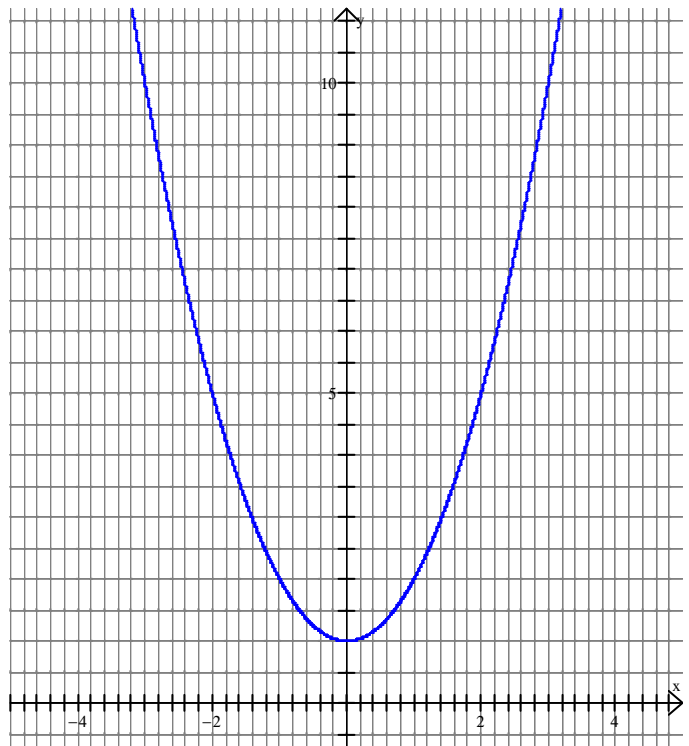
6. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 + 1 \\ y = x + 7 \end{cases}$$

Solution

$$y = x + 7$$

x			
y			



∴ From the figure, the point of intersection is _____

∴ The solution of these simultaneous equations is _____

7. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = -x^2 - x + 2 \\ y = x - 3 \end{cases}$$

Solution

$$y = x - 3$$

x			
y			

∴ From the figure, the point of intersection is _____

∴ The solution of these simultaneous equations is _____

8. Solve the following simultaneous equations by graphical method.

$$\begin{cases} y = x^2 \\ y = 8x - 16 \end{cases}$$

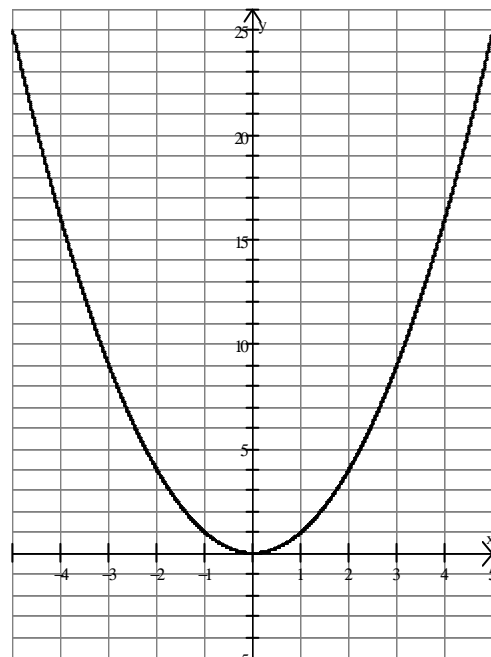
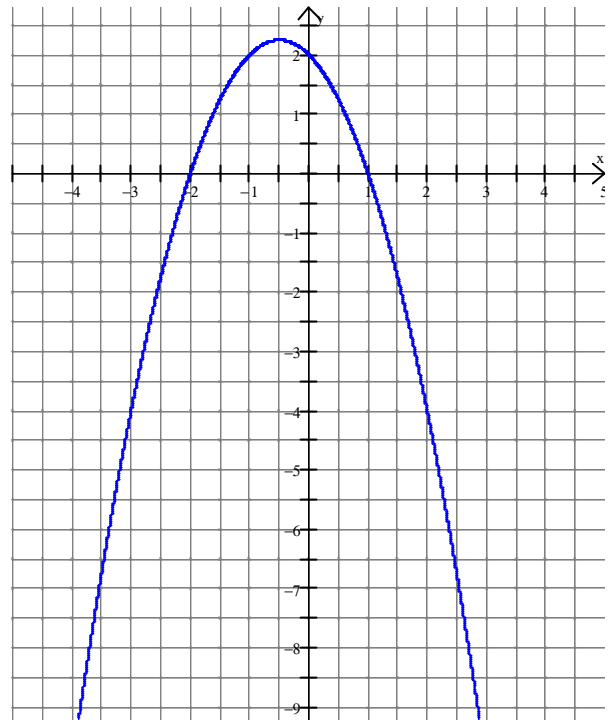
Solution

$$y = 8x - 16$$

x			
y			

∴ From the figure, the point of intersection is _____

∴ The solution of these simultaneous equations is _____



9. Solve the following simultaneous equations by graphical method.

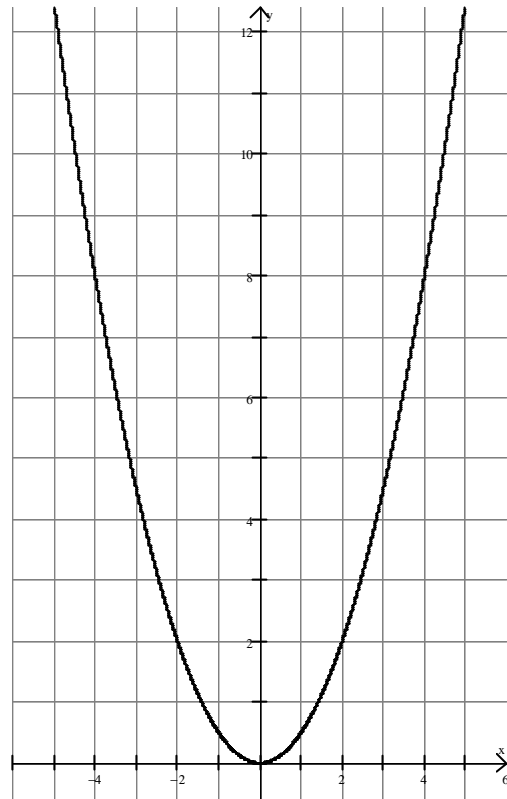
$$\begin{cases} y = \frac{1}{2}x^2 \\ y = \frac{3x}{2} - 2 \end{cases}$$

Solution

$$y = \frac{3x}{2} - 2$$

x			
y			

- ∴ From the figure, we know the point of intersection is _____
- ∴ The solution of these simultaneous equations is _____



Appendix 3

Mathematics Test

Coordinates and Solving Linear Equations in Two Unknowns by Graphical Method

Name: _____ () Class: _____ Score : _____

Date: _____ Total: 64

Time: 35 minutes

1. Write down the coordinates of points A to F in the rectangular coordinate plane shown in the diagram.

The coordinates of A are = _____

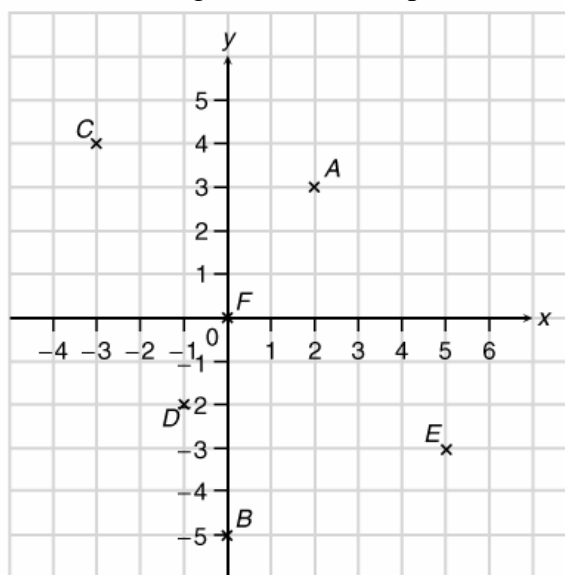
The coordinates of B are = _____

The coordinates of C are = _____

The coordinates of D are = _____

The coordinates of E are = _____

The coordinates of F are = _____



(6 marks)

2. With reference to the given linear equations in two unknowns, complete the corresponding tables respectively.

(a) $y = -5x$

x	-1		
y		10	-20

(b) $3x + y = 5$

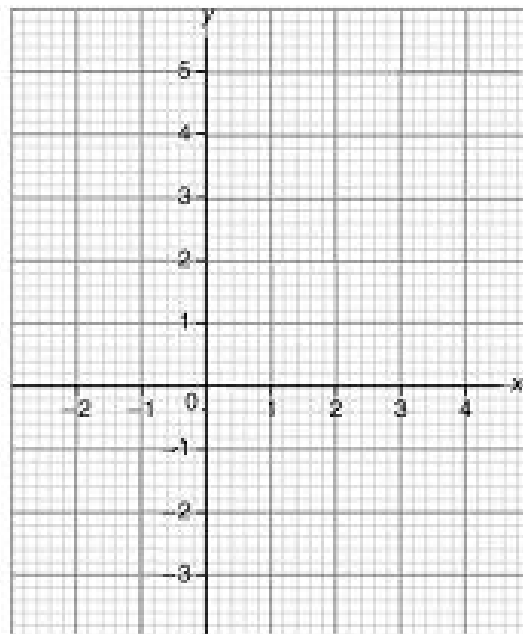
x	-2		2
y		0	

(12marks)

3. Complete the following table, and draw the graph representing the linear equation in two unknowns $4x - 3y = 1$.

Solution?

x	-2	1	4
y			



(8marks)

4. The given figure shows the graph of the linear equation in two unknowns $x - y = 3$.

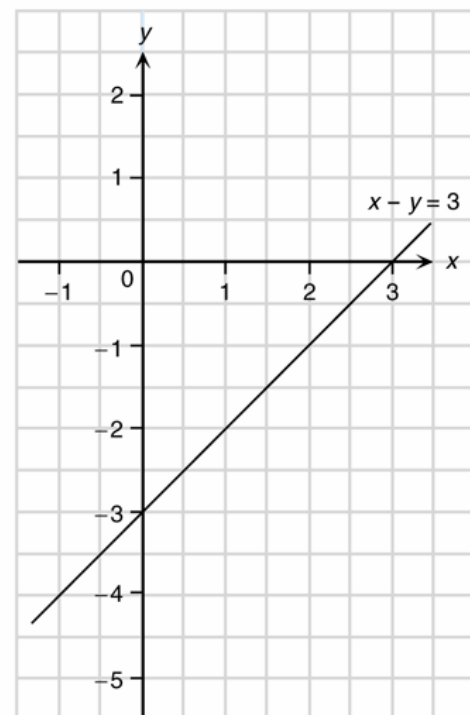
- (a) (i) With reference to the linear equation in two unknowns $3x + y = 1$, complete the following table.

x	0	1	2
y			

- (ii) Try to draw the graph representing the linear equation in two unknowns $3x + y = 1$ in the figure on the right.

- (b) Use the results of (a) and graphical method to solve

$$\begin{cases} x - y = 3 \\ 3x + y = 1 \end{cases}$$



(18 marks)

Solution?

5. Solve the following pair of simultaneous linear equations in 2 unknowns graphically.

$$\begin{cases} 2x + 3y = 5 \\ 3x - y = 2 \end{cases}$$

(20 marks)

Solution?

With reference to the linear equation in two unknowns $2x + 3y = 5$, complete the following table.

x	-2	1	4
y			

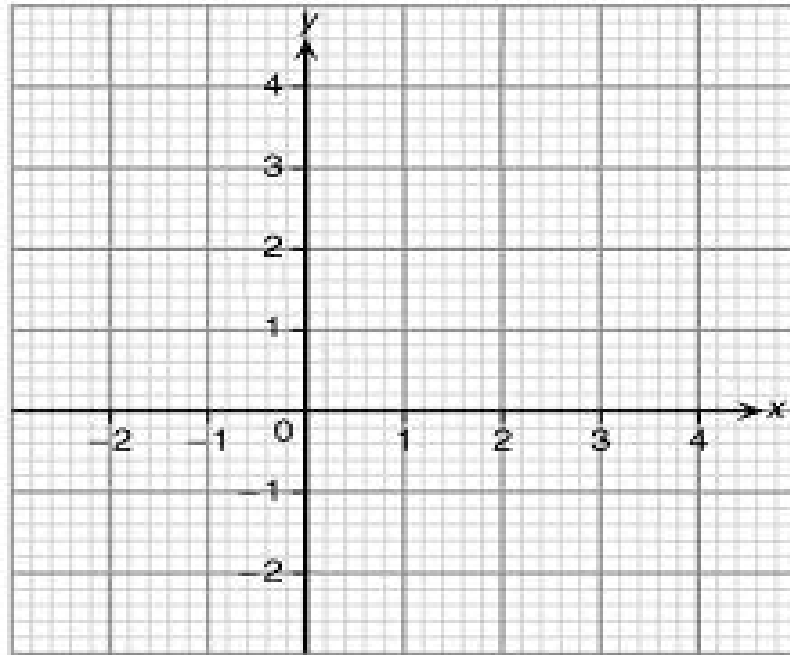
With reference to the linear equation in two unknowns $3x - y = 2$, complete the following table.

x	0	1	2
y			

Draw the graph representing the pair of linear equations in two unknowns $2x + 3y = 5$ and $3x - y = 2$ in the diagram below.

From the graph, the two straight lines intersect at the point (,).

∴ The solution of the system of linear equations in two unknowns is _____



End